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ON THE RECOVERY OF TRUE TEMPERATURE FROM THERMAL RADIATION SPECTRUM: COMPUTER EXPERIMENT

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I. INTRODUCTION

It is known that intensive energy fluxes affecting an object can change its aggregate state, i.e., melting, boiling, and partial evaporation of an object during subsecond time intervals may occur. So, at every moment, the thermal radiation spectrum contains unique information about the object state. The spectral intensity (brightness) $I(\lambda)$ from the area element at the wavelength λ is registered in the sighting direction by a pyrometer (or spectrometer). For a free-emitting surface, there is no reflected radiation, and the thermal radiation spectrum is determined by temperature T and by optical properties of the surface. Therefore, the following equality is true:

$$I(\lambda) = \varepsilon(\lambda, T)\, I_0(\lambda, T), \qquad (1)$$

where $\varepsilon(\lambda, T)$ is the spectral radiant emittance (or spectral emissivity) of the sighting area in the sighting direction, and $I_0(\lambda, T)$ is the spectral intensity of a blackbody at temperature T and wavelength λ, calculated in accordance with Planck formula:

$$I_0(\lambda, T) = \pi^{-1} C_1 \lambda^{-5} \left[\exp\left(\frac{c_2}{\lambda T}\right) - 1\right]^{-1}.$$

Here C_1, c_2 are the known constants. Functions $I(\lambda)$ and $\varepsilon(\lambda, T)$ are assumed to be continuous. To determine uniquely the temperature T from the spectral intensity $I(\lambda)$ we have to solve the equation (1) with value $\varepsilon(\lambda, T)$, which is often unknown. Without knowledge of the radiant emittance $\varepsilon(\lambda, T)$, the problem of contactless measurement of the temperature of a free–emitting body is underdetermined. Thus, the problem is ill-posed with infinitely many solutions.

It is well known that any additional information about the solution is of great importance for solving ill posed problems [1]. By including the information in the problem statement and in solving algorithms, we can pick up the solution with required a priori properties and make the problem numerically stable.

To determine the temperature from equation (1) with unknown function $\varepsilon(\lambda, T)$, we accept following general a priori assumptions:

a) parametric form of function is given: $\varepsilon(\lambda) = \varepsilon(\lambda, \mathbf{a})$ where $\mathbf{a} = (a_1, a_2, ..., a_n)$ is the vector of unknown parameters that have to be found together with T from the system of m equations

$$I(\lambda_i) = \varepsilon(\lambda_i, \mathbf{a})\, I_0(\lambda_i, T), \; i = 1, 2, ..., m, \qquad (2)$$

with $m > n + 1$.

b) a priori intervals for T and λ are given: $T \in [T_{\min}, T_{\max}]$, $\lambda \in [\lambda_{\min}, \lambda_{\max}]$.

The examples of those assumptions in different combinations can be found in papers [2–5]. In this paper we present a numerical algorithm for determination of T and $\varepsilon(\lambda)$, which is based on the minimization problem for the function of discrepancy. Using a special form of initial approximation, the algorithm specifies consequently the type of relationship $\varepsilon(\lambda_i, \mathbf{a})$, proceeding from the simplest linear to more complex dependencies, being nonlinear in λ and $\mathbf{a}$. In doing so, the corresponding temperature is adjusted for every new dependence. The process continues until we obtain that the calculated relationship $\varepsilon(\lambda_i, \mathbf{a})\, I_0(\lambda, T)$ is adequate to the experimental data. The algorithm was tested for a wide class of data generated with the help of model values $\varepsilon(\lambda)$. The algorithm stability was estimated under data disturbance with model random errors, being similar to experimental errors in the magnitude. The proposed algorithm allows to estimate a range of possible recovered temperatures and corresponding standard deviation.

II. BASIC RELATIONSHIPS

Data $I(\lambda)$ are given on the grid $\{\lambda_i\}_{i=1}^m$, $\lambda_i \in [\lambda_{\min}, \lambda_{\max}]$, with an experimental error. So, we know, instead of $I(\lambda_i)$, approximate values $I^\delta(\lambda_i)$, measured with δ r.m.s error

$$\delta = \left(\sum_{i=1}^m \left[I(\lambda_i) - I^\delta(\lambda_i)\right]^2\right)^{1/2} \Big/ S,$$

$$S \equiv \left\{\sum_{i=1}^m \left[I^\delta(\lambda_i)\right]^2\right\}^{1/2}.$$

Therefore, accepting specific analytical model $\varepsilon(\lambda) = \varepsilon(\lambda, \mathbf{a})$ (with parameters $\mathbf{a}$ from the permissible class D), we just can try to fit adequately the input data $I^\delta(\lambda_i)$ using the relationship $I(\lambda; \mathbf{a}, T) = \varepsilon(\lambda, \mathbf{a})\, I_0(\lambda, T)$. The relative accuracy of fitting is characterized by the value of discrepancy:

$$\Delta(\mathbf{a}, T) = \frac{1}{S^2} \sum_{i=1}^m \left[\varepsilon(\lambda_i, \mathbf{a})\, I_0(\lambda_i, T) - I^\delta(\lambda_i)\right]^2.$$

The best accuracy of fitting (in the sense of Least Squares) should provide the optimal values T^* and $\mathbf{a}$ for which the following is fulfilled:

$$\Delta(\mathbf{a}^*, T^*) =$$

$$\inf\{\Delta(\mathbf{a}, T) : \mathbf{a} \in D, T \in [T_{\min}, T_{\max}]\} \equiv \delta^2_{\text{appr}}. \qquad (3)$$

Minimizing the function $\Delta(\mathbf{a}, T)$ in arguments $\mathbf{a}$ and T, we obtain the value of δ_{appr} as a characteristic of the accepted model $I(\lambda; \mathbf{a}, T)$. The model is considered to be adequate for data $I^{\delta}(\lambda_i)$, if $\delta_{appr} \leq \delta$. Below we refer to the number δ_{appr} as the accuracy of data approximation by accepted model. Description of the algorithm for problem solution is given in [6].

III. RESULTS OF COMPUTATIONAL EXPERIMENTS

To analyze the properties of presented algorithm, we tested it in a series of computational experiments with data obtained in a scheme of "quasi-real" experiment [1].

In a real experiment, the values $I^{\delta}(\lambda_i)$ are found with the error depending on random and systematic factors. Besides, the values of used wavelengths are known only approximately. In a "quasi-real" computational experiment, the wavelengths λ_i are given exactly, and sighting supposed to be normal to the area element. The "quasi-real" experimental data were calculated for a chosen substance by formula $I^{\delta}(\lambda_i) = \varepsilon(\lambda_i, T_0) I_0(\lambda_i, T_0)$. Here $\varepsilon(\lambda_i, T_0)$ is the experimental normal emissivity at a known temperature T_0. Quasi-real data $I^{\delta}(\lambda_i)$ were disturbed by normal random error with the zero mean and the relative standard deviation δ (see details in [7]). Below, some results of our computations for model temperature recovery are presented in fig.1 and 2. The figures illustrate the minimization procedure (3). The curves present for different error levels δ calculated best residual values for parametric dependencies, i.e.

$$\Delta(T) = \inf\{\Delta(\mathbf{a}, T) : \mathbf{a} \in D\},$$

depending on T, while the numbers δ are depicted by horizontal continuous lines. Crossections of lines and curves determine intervals of true temperature estimates T_{calc}.

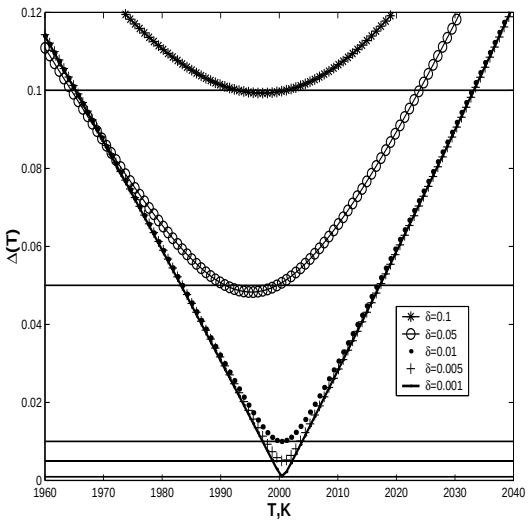

Figure 1. Illustration of T_{calc} finding for Molybdenum at different r.m.s errors δ. True temperature is $T_0 = 2000K$ (the emissivity data from [8]).

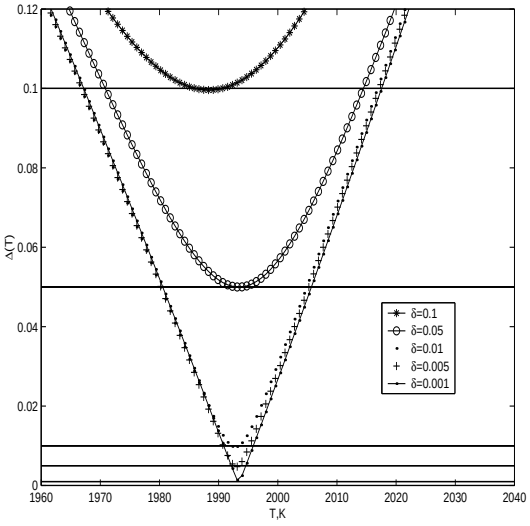

Figure 2. Illustration of T_{calc} finding for Tantalum at different r.m.s errors δ. True temperature is $T_0 = 2000K$ (the emissivity data from [9]).

IV. CONCLUSIONS

1. The algorithm of temperature and emissivity dependence recovery from continuous spectra of emitted radiation (for opaque heated bodies) was developed for a wide class of parametric models of $\varepsilon(\lambda, \mathbf{a})$, which are typical for real materials. This algorithm allows an estimate of a range of possible temperatures and corresponding snandart deviation.

2. The database of parametric models $\varepsilon(\lambda, \mathbf{a})$ can easily be modified or extended for different types of the studied object and/or its surface structure.

3. For an isothermal system of surfaces (or cavities), the temperature of the sighting area in a given direction can be found using the same algorithm.

4. Proposed algorithm allows to estimate ranges of possible true temperatures for given levels of data error δ. The ranges appear to be "narrow" enough to estimate true temperature with high accuracy.

5. The influence of increasing number of terms n in parametric dependencies $\varepsilon(\lambda, \mathbf{a})$ on the accuracy of true temperature recovery was investigated. It was found for examples of polynomial parametrical dependencies, that if $n > 10$ the recovery problems appear to be highly ill-conditioned. So, reliable temperature estimates can be obtained when $n \sim 1 - 4$.

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