Investigation of Functions $W(T_{90})$ for Low-α PRTs in the Sub-ranges above 0 °C

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Abstract. The most practical method of interpolation for industrial platinum thermometers is the use of the Callendar-Van Dusen second-order equation. Standard tables R(T) for IPRTs of different purity of the platinum wire were established by number of Standards (IEC, DIN, GOST, JIS). From fitting the coefficients B vs. A of the CVD equations, obtained for 166 thermometers with W(100) ranging from 1.380 to 1.392, the linear function B(A) was generated in this work, which makes it possible to establish the second-order reference function for PRTs of any nominal W(100) value. It is important, that substantial distortion of the interpolating CVD curves was found for the IPRTs with film platinum sensing elements. Although the ITS-90 interpolation method is supposed to be applied only to the PRTs, that have a strain-free sensing element made of platinum wire of a very high purity (W(Ga)> 1.11807), it has become usual practice to use the ITS-90 function for industrial thermometers. As shown in the paper, a systematic difference occurs between temperature values calculated by means of the CVD equation and the ITS-90 interpolation technique, which does not depend on the purity of the platinum wire in a large W(100) range, but highly depends on the temperature sub-range. For individual calibration of an IPRT in the sub-range 0-230 °C, it is possible to use only one calibration point, besides 0 °C.

INTRODUCTION

Accuracy of approximation of the Thermodynamic Temperatures by the International Temperature Scale ITS-90 depends to some extent on the form of the *W(T)* equation stated in the Scale. According to the theory, dependence of the electrical resistance of a platinum thermometer on the temperature in the range above 0 °C can be well described by a second-order polynomial. However, consideration of some complicated processes going in the platinum, leads to adding components of higher degrees to the resistance-versus-temperature function. The accurate measurements made with a high-pure HTSPRT in the frames of the work on developing ITS-90, revealed a ninth-order function $W_{ref}(T)$, that was established as the reference function of the Scale in the range from -13.8 to 1234.93 K. Individual function *W(T)* of a PRT is determined as a sum of $W_{ref}(T)$ and $\Delta W(T)$, the deviation function, calculated from the results at the fixed points of the scale. One of the most important characteristics of a PRT, which the value of the deviation function depends on, is the purity of the platinum wire of the PRT sensing element. The restriction on the purity of the platinum in ITS-90 was set by means of the resistance ratio at the gallium fixed point: $W(Ga) \geq 1.11807$.

A lot of IPRTs in industry and science have a *W(Ga)* smaller than the lower limit set by the ITS-90 for SPRTs. Since the achievable accuracy of IPRTs is worse than that of SPRTs, methods of their calibration are supposed to be cheaper, the interpolation equations should be simpler than those for SPRTs. Usually, for a particular purity of platinum, which is defined in terms of *W(100)=R(100)/R(0)*, one reference table *W(T)* is established for the application to any PRT of this nominal ratio *W(100)*. Thus, for IPRTs with *W(100)*=1.385 the coefficients of the Callendar - Van Dusen second-order equation were derived in [1] and the reference table was established in Standard IEC-751, the tolerances for IPRTs of Class A being as large as ±0.15 °C at 0 °C. The coefficients and the reference table for IPRTs with *W(100)* = 1.391 were established in GOST 6651 and OIML R84. In Japanese Industrial Standard a forth-order function was established for thermometers JPt-100 with 1.392 [3]. A fifth-order reference function was developed for IPRTs of different purity in [2]. In the present work we analyze several reference functions and results of calibrations of a large group of PRTs with the ratios varying from *W(100)*=1.3798 to *W(100)*=1.3928. An attempt to reveal the actual relationship between the coefficients of the CVD second order polynomial was made.

Recent research works with IPRTs showed that some IPRTs allow measuring temperature with an uncertainty smaller than the tolerances stated for IPRTs in the IEC document. For such thermometers it became a common practice to use the ITS method, ignoring the restriction on *W(Ga)*. It was found in [4], [5], that a systematic difference occurred between interpolation functions calculated using the ITS-90 equations and the CVD second-order polynomial. The difference does not depend on *W(100)*, but depends on the temperature range. For the ranges 0-160 °C and 0-230 °C the error is within ±0.01 °C. The individual CVD function for an IPRT can be calculated as a sum of the quadratic approximation of the ITS-90 reference function in this range, and a linear deviation function.

ANALYSIS OF THE RELATIONSHIP BETWEEN THE COEFFICIENTS OF THE CVD EQUATION

Calibration data of more than 160 thermometers were used in this work. Some thermometers were investigated by other scientists and the data were published in [1], [2], [3]. Crovini et al. [1] made a research with wire-type IPRTs and derived a reference function W(T) by averaging the data of twelve best thermometers with W(100) of about 1.385. The difference between two calibrations of those thermometers in the range 0-630 °C was from 6 to 180 mK. The largest irreproducibility was observed for the following PRTs: 05M – 180 mK, 01H – 167 mK, 05H – 139 mK, 04M – 87 mK. The largest hysteresis at 550 °C was found for the following PRTs: 05H – 110 mK, 06H – 167 mK, 04H – 70 mK. Zhang Jipei et al. [2] investigated and calibrated 35 IPRTs of five different designs including thick-film elements. The thermometers were manufactured by six different companies. The best stability was observed for ceramic sensing elements with the upper temperature limit of 500 °C. Change in the resistance during the stability tests was not greater than 0.05 °C equivalent for about 70% of the thermometers. Sakurai et al. [3] carried out stability tests and calculated a reference function for JPt100 thermometers specified by the Japanese Industrial Standard. The short-term stability of some thermometers under study was better than 10 mK in the range 0-500 °C.

More than 90 IPRTs were tested and calibrated at VNIIM last years. All the thermometers contained wire-in-ceramic sensing elements manufactured by three Russian companies. *W(100)* values of the thermometers were greater than 1.391. Initially the PRTs were stabilized at an annealing furnace at 675 °C for 80 hours. Then the thermometers were calibrated at the fixed points of Sn, Zn, TPW. The calibration cycle was repeated three times. Irreproducibility of W(FP) values at the fixed points was better than 0.04 °C equivalent for more than 80% of the IPRTs. Several thermometers of AMETEK, which are used with DTI-100 devices, were also calibrated at the fixed points at VNIIM. *W(100)* values of these IPRTs were about 1.385. To get more information on PRTs of different purity, we also used in the study calibration results of several SPRTs and HTPRTs.

Nine special PRTs were manufactured at VNIIM using platinum wire of different purity. Some metal impurities were intentionally added to the pure platinum, thus, making *W(100)* to vary from 1.379 to 1.391.The design of the sensing elements was strain-free (Strelkov design). The thermometers were calibrated at the fixed points: TPW, In, Sn, Zn. Some of these PRTs were investigated previously in work [4]. The information on the thermometers involved in the present study is summarized in Table 1.

TABLE 1. PRTs Under Investigation

Organization, Reference	Type of PRTs,	Number of PRTs	Number of Manufactures	Range of W(100)	Calibration
VNIIM	IPRT	95	3	1.3915 - 1.3924	Fixed points
VNIIM	Special PRT	9	1	1.3798 – 1.3915	Fixed points
VNIIM	IPRT-AMETEK	5	1	1.3840 – 1.3860	Fixed points
VNIIM	SPRT, HTPRT	10	1	1.3925 – 1.3927	Fixed points
IMGC [1]	IPRT	12	several	1.3845 – 1.3861	Calibration bath
SIPAI [2]	IPRT	35	6	1.3843 – 1.3858	Calibration bath
Total number of PRTs		166			

We calculated the coefficients of the CVD equations for all the PRTs described above. In Fig.1 the graph of the dependence of *B* on *A* is presented. On the right side of the graph one can see two groups of points overlapping each other. They refer to 95 IPRTs and 10 SPRTs calibrated at VNIIM. The points on the left side of the graph present the results of the PRT calibrations taken from papers [1], [2]. Dark triangles, that are markers for 9 special thermometers, are spread throughout the graph, from low to high coefficients *A*. Big markers represent the reference functions. Fifth and forth order polynomials from [2] and [3] were approximated by the CVD functions.

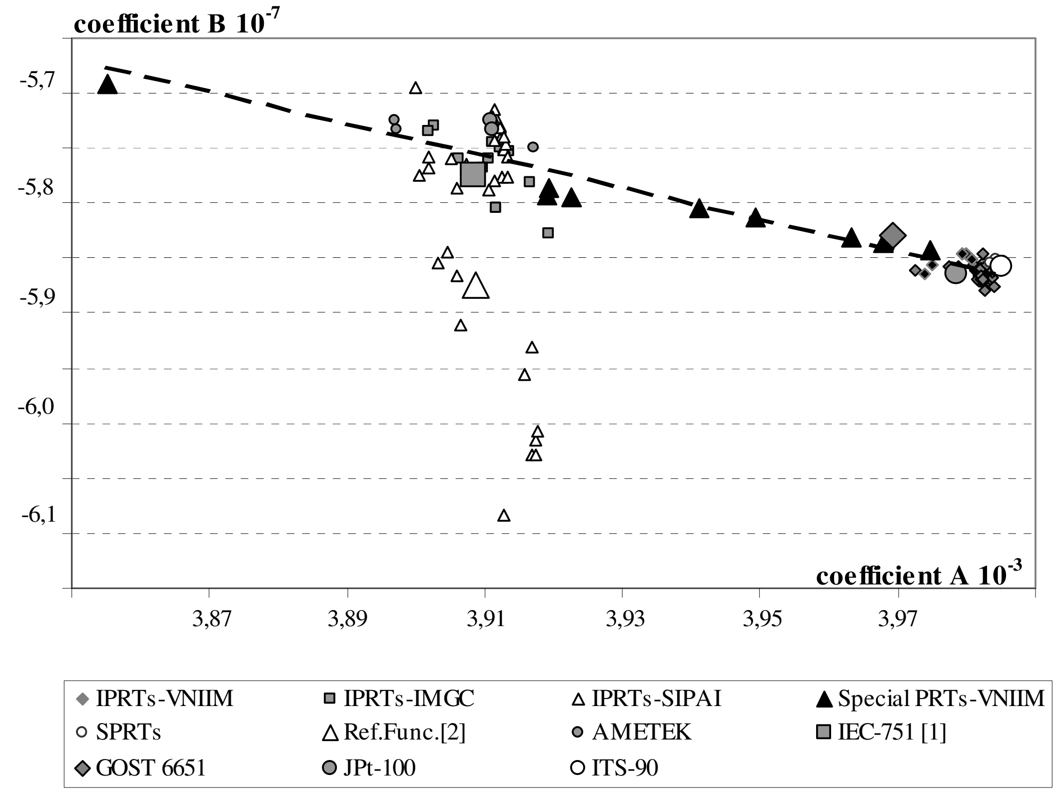

FIGURE 1. The relationship between *B* and *A* for different PRTs.

It can be noticed from the graph that the data for PRTs with higher *A* values are less scattered than those for low-*A* PRTs. It may be related to some changes in the platinum due to impurities, which cause the instability and inaccuracy of the calibration. Also, it can be related to the fact, that the PRTs in [1] and [2] were calibrated in temperature controlled bathes, while the PRTs at VNIIM were calibrated at the fixed points. Some points that represent thermometers from [2], lay much lower than the other points. Investigating this phenomenon we noticed, that all those low results were obtained with thick-film thermometers. Paper [2] claims a good stability for the thick-film thermometers. Probably, there is a difference between the properties of thick film and wire platinum, which causes the change in the coefficients of the *W(T)* equation. Averaging the data for all the thermometers in [2] gave the reference function with the coefficients that remarkably deviate from the coefficients of the other IPRT reference functions. It can be suggested that thermometers with film sensing elements usually have lower ratios *B/A* and a larger range of the ratios. We do not have results on calibration of some other film thermometers in the range 0-420 °C. Some experience in working with the film IPRTs used in heat meters in the range 0-160 °C [6] confirmed the decrease in the values of ratios *B/A*. All the data on Fig.1, excluding the thick film thermometers and SPRTs, were pooled together, and a graph *B(A)* was obtained by the least-square method. The graph obtained is a line with ratio $B/A = -1.4678\times10^{-4}$. It should be noted that the lowest points from the group of results taken from the paper [2], refer to thermometers 05H and 06H, which exhibited the largest hysteresis during the calibration. The data for the most stable thermometers are in a good agreement with the line. One of the observations from the graph is a lower position of some points representing the data of the special thermometers with A of about 1.392 10^{-3}. The residuals of the data from the line fit are shown in Fig.2.

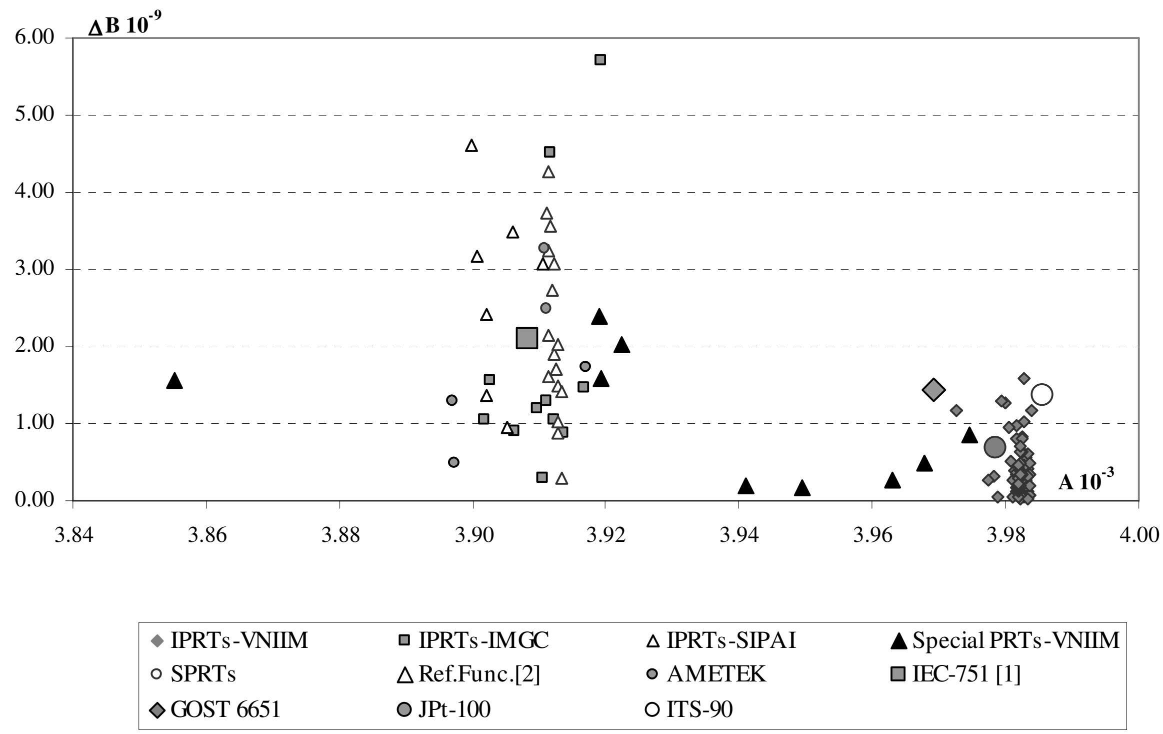

FIGURE 2. The residuals of the experimental data from the calculated line.

The maximum values of the deviation for the low-A thermometers are smaller than 6×10^{-9}. The residuals for the high-A thermometers are not greater than 2×10^{-9}. The uncertainty of the coefficients depends on the uncertainties of the calibration and instability of the thermometers. The application of the low of uncertainty propagation shows that if the uncertainty of the $W(T)$ values (which includes instability and hysteresis) is larger than 0.05 °C, then we are not able to determine B with an uncertainty smaller than 5.7×10^{-9} , that is within the range of deviations of the experimental data from the calculated linear function in Fig. 2.

Thus, the $B(A)$ dependence estimated in this work, may be used for defining the quadratic reference functions for wire PRTs with different $W(100)$ values. The difference between coefficients B of ITS-90, IEC-751, JPt-100, GOST-6651 functions and the calculated coefficients are subsequently: 1.37×10^{-9}, -2.11×10^{-9}, -0.68×10^{-9}, 1.43×10^{-9}. The value for JPt-100 function is the closest to the theoretical value.

The constant value of B/A may also be interpreted as a possibility for a linear deviation function to be applied to the quadratic reference function for developing the interpolation equation for a PRT. Actually, we can take a quadratic CVD approximation of ITS-90 function in the range 0 - 420 °C as the reference function for IPRTs,

$$W_{r90}(T) = 1 + A_{90}\, T + B_{90}\, T^2 \qquad (1)$$

where

$$A_{90} = 3.9856\times10^{-3}\ °C^{-1}\ ;\ \ B_{90} = -5.8536\times10^{-7}\ °C^{-2},$$

and calculate the deviation function as

$$\Delta W(T) = a\,(W_{r90}(T) - 1), \qquad (2)$$

where a is calculated from the result of calibration at Zn point:

$$a = [W(Zn) - W_{r90}(Zn)]/[W_{r90}(Zn)-1]. \qquad (3)$$

If $\quad W(T) = 1 + A_1\, T + B_1\, T^2 , \qquad (4)$

then $A_1 = (1 + a) A_{90}$; $B_1 = (1 + a) B_{90}$ (5)

Thus, $B1/A_1 = B_{90} /A_{90}$

The ratio B_{90}/A_{90} is very close to B/A of the theoretical line in Fig. 1. The linear deviation function is a direct sequence of the Matthiessen rule for pure platinum, which states that the electrical resistivity of platinum is a superposition of the phonon component and the component resulting from the lattice defects. The Matthiessen rule works well at low temperatures. The higher the temperature and the impure the platinum, the worse it works. However, the restricted accuracy of IPRT allows using this physical law even in the range 0-420 °C.

THE SYSTEMATIC DIFFERENCE BETWEEN CVD AND ITS-90 EQUATIONS

For the most of IPRTs used in industry, the tolerances of several tenths of kelvin are quite satisfactory. However, there are some IPRTs, which are proved to have a better stability and are intended to be used in calibrators and baths for controlling the temperature and as reference standards. Such PRTs are usually used with digital devices, which convert the resistance in the temperature by means of a quadratic function with individual coefficients. When we calibrate an IPRT against a SPRT in a calibration bath and calculate the temperature of the IPRT using CVD function and the temperature of the SPRT using ITS-90 function we make a systematic error, which is caused by the difference between the second and ninth order functions. This error strongly depends on the range. Marcarino et al. in [5] investigated the error function ΔT(T) for a large number of PRTs with W(100) values of about 1.392. He found that the value of the error does not depend on *W(100),* but depends to a great extent on the temperature range. It was suggested to apply a correction function to *W(T).* However, that function seems to be very complicated for wide using among the customers of IPRTs. In this work we tried to estimate the deviation of the CVD function from the ITS-90 function for PRTs of a large *W(100)* range. For the investigation we chose several thermometers described in Table 1. Among them were three IPRTs with *W(100)* of about 1.392, two IPRT with *W(100)* of about 1.385, three SPRTs, and nine PRTs of the special design. The curves of the deviation were found to lie very close to each other. It would be difficult to show them in one graph. In Fig.3 the deviation of the ITS-90 reference function from its quadratic approximation is shown as a function of temperature for two temperature ranges.

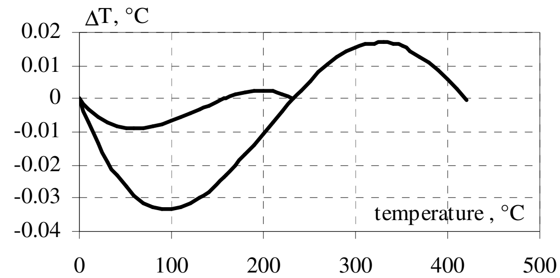

FIGURE 3. The deviation of the ITS-90 reference function from its quadratic approximation

The maximum deviations from the curves in Fig.2 for all the thermometers under investigation are shown in Table 2. Although the SPRTs exhibited the smallest deviations, all the values in Table 2 are very small as compared with the uncertainties achievable for industrial thermometers. For IPRTs, we can conclude that the correction to the CVD function does not depend on the purity of platinum in a large *W(100)* range.

TABLE 2. Deviations of CVD Functions for Different PRTs From the Curves in Fig.3 in Maximums, mK

		0 – 420 °C		0 – 232 °C	
PRT S/N	**W(100)**	**100**	**330**	**60**	**200**
SPRT 4185	1.3927	0.01	0.01	0.03	0.01
SPRT 74	1.3926	0.03	0.02	0.03	0.01
SPRT 012	1.3927	0.05	0.04	0.04	0.01
IPRT V-1	1.3921	0.31	0.31	0.05	0.03
IPRT V-5	1.3920	0.35	0.35	0.10	0.05
IPRT V-9	1.3915	0.33	0.30	0.12	0.04
IPRT L-3	1.3854	1.23	0.80	0.36	0.10
IPRT L-7	1.3854	0.94	0.59	0.38	0.08
PRT 1	1.3861	2.04	0.30	0.35	0.09
PRT 2	1.3865	2.58	0.98	0.36	0.09
PRT 3	1.3798	0.15	0.26	0.38	0.08
PRT 4	1.3891	0.18	0.17	0.12	0.03
PRT 5	1.3909	0.20	0.16	0.10	0.02
PRT 6	1.3916	0.11	0.09	0.13	0.04
PRT 7	1.3905	0.25	0.21	0.14	0.03
PRT 8	1.3883	0.26	0.24	0.16	0.04
PRT 9	1.3862	0.62	0.53	0.30	0.07

IPRTs are often used above 0°C in rather narrow ranges. Examples of such use are – PRTs for heat meters (0-160 °C), water baths (0-100 °C), and oil baths (100-250 °C). An advantage of the use is a better stability of PRTs. Considering that in the sub-ranges up to 230 °C the correction to the ITS-90 function is smaller than 10 mK in maximums, we can suggest that the second-order CVD function may be used there quite successfully. Approximating ITS-90 function in the ranges 0-156 °C, 0-230 °C by CVD equation we will get different coefficients *A* and *B*. The ratios *B/A* will also be different.

In sub-range 0-156 °C

$A_{90} = 3.9881\times10^{-3}\ °C^{-1}$, $B_{90} = -5.9827\times10^{-7}\ °C^{-2}$.

In sub-range 0-230 °C

$A_{90} = 3.9873\times10^{-3}\ °C^{-1}$, $B_{90} = -5.9300\times10^{-7}\ °C^{-2}$.

Applying the linear deviation function (2) calculated from the calibration at one temperature point to the quadratic reference function, one can obtain the individual interpolation equation for a PRT in these ranges. We investigated accuracy of the application of the linear deviation function calculated from the result at Sn point to the quadratic approximation of ITS-90 in the range 0-230 °C. The curves in Fig.4 presents the difference between the individual functions obtained for each PRT and the functions calculated using the ITS-90 interpolation technique.

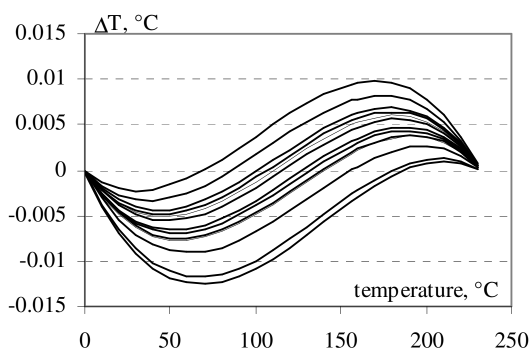

FIGURE 4. The deviation of the interpolation functions calculated using the CVD approximation of the ITS-90 reference function and the linear deviation function from the function obtained using the ITS-90 interpolation method.

The interpolation uncertainty of the method is smaller than ±0.015 °C for all the thermometers under study. The advantage of this technique is simplicity of the calibration and calculation procedures and a low cost of the calibration.

CONCLUSIONS

It is possible to develop a standard table for wire-type IPRTs with different nominal *W(100)* values using ratio *B/A* of the coefficients of the quadratic approximation of ITS-90 for a particular temperature sub-range.

The systematic difference occurs between the temperatures calculated by using of the CVD equation and by the ITS-90 method. This difference does not depend on the purity of the platinum wire in the large *W(100)* range from 1.379 to 1.327. For precise calibration of low-a wire PRTs in the sub-ranges below 230 °C it is possible to use only one calibration point, besides 0 °C. The difference between temperatures calculated using the ITS method and the method suggested in the paper does not exceed ±0.015 °C.

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