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Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI-2005)

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Abstract

The International System of Units (SI) is founded on seven base units, the metre, kilogram, second, ampere, kelvin, mole and candela corresponding to the seven base quantities of length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity. At its 94th meeting in October 2005, the International Committee for Weights and Measures (CIPM) adopted a recommendation on preparative steps towards redefining the kilogram, ampere, kelvin and mole so that these units are linked to exactly known values of fundamental constants. We propose here that these four base units should be given new definitions linking them to exactly defined values of the Planck constant h, elementary charge e, Boltzmann constant k and Avogadro constant N_A, respectively. This would mean that six of the seven base units of the SI would be defined in terms of true invariants of nature. In addition, not only would these four fundamental constants have exactly defined values but also the uncertainties of many of the other fundamental constants of physics would be either eliminated or appreciably reduced. In this paper we present the background and discuss the merits of these proposed changes, and we also present possible wordings for the four new definitions. We also suggest a novel way to define the entire SI explicitly using such definitions without making any distinction between base units and derived units. We list a number of key points that should be addressed when the new definitions are adopted by the General Conference on Weights and Measures (CGPM), possibly by the 24th CGPM in 2011, and we discuss the implications of these changes for other aspects of metrology.

1. Introduction

1.1. Overview and background

In devising the definition for a base unit, the most important quality that one strives to achieve is that the particular quantity used to define the unit should be a true invariant of nature, invariant under translation in space and time—even on an astronomical scale. In this respect, it is understood that in the framework of general relativity, units are defined as '*proper units*; they are realized from local experiments in which the relativistic effects that need to be taken into account are those of special relativity' [1]. Other qualities of a more practical nature are perhaps less important, but include the following. The practical realization of the definition of a base unit should, in principle, be possible anywhere, at anytime and as accurately as the best practical measurements require, although it is recognized that the advanced metrology needed may call upon considerable intellectual and financial resources. Practical realizations of the definitions of base and derived units should be easily accessible to workers in all areas of science and technology so that one can be confident that in complex fields, for example, in global climate studies, data from widely different scientific and technological areas are based on consistent units. Further, since it is important that the basis of our measurement system be taught in schools and universities, it is preferable, as far as modern science permits, that the definitions of base units be comprehensible to students in all disciplines, a requirement that becomes increasingly difficult to achieve as science advances. Finally, if the definition is to replace an earlier definition of the same unit, it should be chosen to preserve continuity, so that the new definition should be consistent with the previous definition within the uncertainty that the previous definition could be realized.

The desire to use true invariants of nature, which we take to be the fundamental constants of physics or the properties of atoms, as reference quantities in practical measurements and for defining units has led to the development of a new subject often called ‘quantum metrology’. The original definitions of the metre and the kilogram from the 18th century, as well as the much older unit of time, the second, were made in terms of the dimensions of the Earth and its period of rotation, but as James Clerk Maxwell observed in 1870 [2], these are not true invariants, since ‘the properties of our planet can change and it would still be our planet, but if the properties of an atom were to change it would no longer be the same atom’. To the properties of atoms one would now add, of course, the fundamental constants of nature. At the time, however, existing technology and the state of knowledge of science did not allow Maxwell’s precept to be implemented, and thus the Metre Convention of 1875 chose to establish new prototype artefacts for the metre and the kilogram to be kept at the International Bureau of Weights and Measures (BIPM). The 1st General Conference on Weights and Measures (CGPM) in 1889 formally adopted the new prototypes as the definitions of these units [1]. However, such artefacts have their problems: it is known that, to a greater or lesser extent, the properties of an artefact change with time. In addition, prototype artefacts cannot be available ‘to anyone, anywhere, at anytime’, being only available for comparison at the laboratory where the prototype is held.

The International System of Units, the SI, is founded on seven base units: the metre, kilogram, second, ampere, kelvin, mole and candela, corresponding to the seven base quantities of length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity, respectively [1]. The present situation with the SI is that, of the seven base units, only the second and the metre are directly related to true invariants, the second being defined in terms of the period of the ground state hyperfine transition frequency in the caesium 133 atom and the metre in terms of the speed of light in vacuum (also making use of the second). The kelvin is specified with reference to a precisely defined thermodynamic state of water which, while certainly an invariant of nature, has a thermodynamic temperature that depends significantly on the impurity content and isotopic composition of the particular water sample used, which complicate and restrict the accuracy with which this definition can be realized. The definitions of the other base units have more fundamental weaknesses. The kilogram is still defined in terms of an artefact, namely, the same prototype sanctioned by the 1st CGPM in 1889, whose mass is known to drift relative to a true invariant, although one cannot say precisely by how much. The weakness of the definitions of the ampere, the mole and the candela derives in large part from their dependence on the definition of the kilogram, although they have other problems which are discussed further below.

The definition of the kilogram is thus central to the problem of improving the SI. Its present definition, adopted by the 3rd CGPM in 1901, reads ‘The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram’. The international prototype, a cylinder with a height and diameter of about 39 mm, is made of an alloy of platinum and iridium with mass fractions of 90% and 10%, respectively, and is kept in a vault at the BIPM in Sèvres, on the outskirts of Paris [3, 4]. Although the international prototype has served well as the unit of mass since it was so designated by the CGPM in 1889, it has one important limitation: it is not linked to an invariant of nature. Thus, the possibility of redefining the kilogram in terms of a true natural invariant—the mass of an atom or a fundamental physical constant—has been discussed during at least the last quarter century. Indeed, the need to replace the current artefact-based definition of the kilogram with one based on such an invariant was recognized in 1999 by the 21st CGPM through its Resolution 7, in which it ‘recommends that national laboratories continue their efforts to refine experiments that link the unit of mass to fundamental or atomic constants with a view to a future redefinition of the kilogram’ [5]. This resolution is in keeping with one of the CGPM’s most important responsibilities: to modify the International System from time to time in order to ensure that it reflects the latest advances in science and technology.

A discussion of the possible redefinition of the kilogram took place during the 93rd meeting of the International Committee for Weights and Measures (CIPM)[1] held in October 2004 [6], stimulated by a note submitted to the CIPM by Quinn (an author of this paper) following discussions in the working group ‘*Unités de base et constantes fondamentales*’ of the French *Académie des Sciences*. As a consequence, the CIPM asked its Consultative Committee for Units (CCU) through the CCU’s president, Mills (also an author of this paper), to study the possibility of having a new, fundamental constant-based definition of the kilogram adopted in the future and to report on the results of its investigation at the CIPM’s 94th meeting in October 2005. Stimulated by this discussion, we published a paper in early 2005 [7] in which we proposed redefining the kilogram using as a reference quantity either the Planck constant or the Avogadro constant and suggested that the redefinition could take place at an early date, namely in 2007 when the 23rd CGPM convenes. This proposal has been widely discussed within the metrology community during the past year, notably at meetings of the CCU, other Consultative Committees (CCs) of the CIPM, by the aforementioned working group of the *Académie des Sciences*, by the Committee on Data for Science and Technology

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[1] For information on the CIPM, its Consultative Committees and the Intergovernmental Organization of the Metre Convention, the reader is referred to the BIPM website www.bipm.org.

(CODATA) Task Group on Fundamental Constants and at a Discussion Meeting held at the Royal Society in London on the Fundamental Constants of Physics, Precision Measurements and the Base Units of the SI [8]. While there was considerable opposition to our suggestion that the kilogram be redefined by the 23rd CGPM in 2007, for reasons we describe later, a broad consensus developed that a new definition for the kilogram could be envisaged for the 24th CGPM in 2011, and that moreover it could be accompanied by changes in definitions of the ampere, the kelvin and the mole.

In this present paper we take account of the many comments, criticisms and suggestions expressed and present a consistent proposal to redefine four of the seven base units of the SI, under the assumption that the requirements set by the CIPM in 2005 are met (see section 1.3 and the appendix). These requirements are that the results of experiments obtained over the next few years are indeed acceptable, all having been agreed with the various CCs and other relevant bodies. We now propose new definitions for the kilogram, the ampere, the kelvin and the mole, using as reference quantities the Planck constant, the elementary charge, the Boltzmann constant and the Avogadro constant, respectively. Possible words for the new definitions are given in section 2, and summarized in table 1. In this way six of the seven base units of the SI would be related to fundamental constants or atomic properties, which are true invariants of nature. Further advantages that would follow from these proposals would be that the values of these four fundamental constants would in future be known exactly in terms of SI units, through mathematical identities (that is, with zero uncertainty), along with the ground state hyperfine transition frequency of the caesium 133 atom and the speed of light in vacuum. The values of a number of other important fundamental constants, or combinations of constants, would also become exactly known in terms of SI units, while many others would be known with greatly reduced uncertainties. Moreover, any possible future changes in the CODATA recommended values [9] of these other constants would be very much smaller than would be the case with the present system[2]. Our general conclusion is that the changes we propose here would be a significant improvement in the SI, which would be of future benefit to all science and technology. We believe that these changes would have the widespread support of the metrology community as well as the broader scientific community and that they could take place at the 24th CGPM in 2011.

In our earlier paper [7], we did not explicitly choose between redefining the kilogram by linking it to an exact value of the Planck constant h or by linking it to an exact value of the Avogadro constant N_{A}. A definition to fix the Avogadro constant has the advantage of conceptual simplicity, since the definition could simply be worded to say that the kilogram is the mass of a specified number of carbon 12 atoms. However, in this paper we specifically advocate defining the kilogram to fix the value of the Planck constant. There are three reasons for preferring this alternative. The first is that if we couple defining the kilogram to fix h with defining the ampere to fix the elementary charge e, as we are now proposing, then both h and e would have exactly defined values, and in consequence both the Josephson constant $K_{\mathrm{J}} = 2e/h$ and the von Klitzing constant $R_{\mathrm{K}} = h/e^2$ would have exactly defined values. This would lead to a simplification and increase in accuracy for all precise electrical measurements, which are now always based on the Josephson and quantum Hall effects. It is important to recognize that in this and all that follows, as in all the current discussions on the redefinition of the base units of the SI, these relations for K_{J} and R_{K} are taken to be exact relations of physics; no current physical theory predicts any correction terms and they have experimentally been found to be universal with respect to the type of sample and a wide range of conditions of use. The second reason is that by defining the kilogram to fix the Planck constant h we are left free to allow the mole to be redefined to fix the Avogadro constant N_{A}, which is also part of our present proposal, as described below. And the third reason for preferring to define the kilogram to fix h is that from the point of view of fundamental physics it plays a more important role than N_{A}: the Planck constant h is the central constant of quantum mechanics, just as the speed of light in vacuum, c_0, is the central constant of relativity, and it is desirable to define our units so that both these constants have exactly defined values.

At its 94th meeting, in October 2005 [10], the CIPM discussed the many developments since its 93rd meeting and adopted Recommendation 1 (CI-2005), entitled ‘Preparative steps towards new definitions of the kilogram, the ampere, the kelvin and the mole in terms of fundamental constants’. In it, the CIPM called for the widest possible publicity to be given to these ideas among the scientific and user communities so that their reactions and views can be taken into account in a timely way. This present paper is a response to the CIPM’s call for a wide discussion to take place and proceeds in the following way.

- In the rest of section 1 we set out the guiding assumptions that we have used in drawing up proposals for new definitions and then give an outline of the requirements for new experimental data needed to proceed with the proposed definitions.
- In section 2 we propose and discuss possible words to implement the new definitions for the kilogram, ampere, kelvin and mole in terms of exactly known values of h, e, k and N_{A}. At the end of this section we suggest a novel way to present such new definitions for the entire SI explicitly in terms of specified constants without associating a particular unit with a particular constant and without making any distinction between base and derived units.
- In section 3 we suggest a number of points that we believe should be addressed at the time the new definitions are formally adopted, presumably by the 24th CGPM when it convenes in October 2011 (the final text of any resolutions to be submitted to the 24th CGPM will, of course, be prepared by the CIPM at its meeting in 2010).
- In section 4 we discuss the impact of the new definitions on both metrology and the fundamental constants.

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[2] Although in some cases the relative uncertainty of the value in SI units of the ratio of certain constants is smaller than that of the SI values of the individual constants themselves, modern science requires that we know the SI values of *all* constants with the smallest possible uncertainties. Two examples of this requirement, briefly discussed in the following paragraph, are the Josephson and von Klitzing constants (see also section 4.2).

- In section 5 we discuss some of the issues involved in preparing practical guides for realizing the four new definitions. Such a guide is known as a *mise-en-pratique* of the definition.
- In section 6 we give a brief conclusion to the paper, and finally, in the appendix, we reproduce the CIPM and CCU recommendations of 2005 that have motivated it.

1.2. Guiding assumptions

The basic assumptions underlying this paper are mainly those underlying the 2005 CIPM and CCU recommendations regarding the redefinition of SI base units (see the appendix). Among the most important of these is that the overall structure of the current SI—that is, the present SI base quantities and their units—should remain unchanged. The reason is that these quantities and units are deemed to meet the current and future needs of both the metrological and scientific communities and are well recognized and understood by the vast majority of the users of the SI throughout the world. Clearly, this assumption precludes consideration of a major restructuring of the SI, for example, replacing mass by energy as a base quantity and making mass a derived quantity, which would lead to the joule becoming a base unit and the kilogram a derived unit, or replacing electric current by charge as a base quantity and making electric current a derived quantity, which would lead to the coulomb becoming a base unit and the ampere a derived unit. It also precludes replacing the name and symbol of the current SI unit of mass, the kilogram, kg, by a new name and symbol with which SI prefixes can be used. For historical reasons, kilogram, kg, contains the SI prefix kilo, k, which means that other SI prefixes cannot be used with either its name or symbol but must be used with the name gram, g [1].

The second assumption is that it is not always necessary that a new definition of an SI base unit should allow the unit to be realized with a reduced uncertainty. In particular, the benefits to both metrology and science of replacing the current definition of the kilogram by one that links it to an exact value of the Planck constant h, and the current definition of the kelvin by one that links it to an exact value of the Boltzmann constant k, are viewed as far outweighing any marginal increase in the uncertainty of the realization of the SI unit of mass or thermodynamic temperature that might result. In fact, the 'uncertainty' of the mass, $m(\mathcal{K})$, of the international prototype of the kilogram with respect to fundamental constants is not well known. As pointed out in [7], although $m(\mathcal{K})$ may vary in time comparatively slowly with respect to the masses of the worldwide ensemble of Pt–Ir standards of about the same age—perhaps by only 50 µg per century—the drift of the entire ensemble relative to an invariant of nature is unknown at a level below 1 mg over a period of 100 or even 50 years.

The third assumption is that the units to be redefined and the constants to which they are to be linked should be chosen in such a way as to maximize the benefits to both metrology and science. This assumption, which is not completely independent of the previous assumption, recognizes a significant problem of the current SI: it has to serve two competing and often conflicting masters. The first is 'everyday commerce', which requires a system of units whose applications range from buying a chicken in the supermarket to building the International Space Station. The second we shall call 'quantum physics', which requires a system of units for determining fundamental constants such as the Planck constant h and the elementary charge e and the properties of the fundamental building blocks of nature such as the mass of the electron m_e and its magnetic moment μ_e. In general, the needs of everyday commerce do not require the smallest possible uncertainties, the exception being perhaps time (commercial satellite navigation systems require a time scale and stable clocks of the highest possible accuracy), but the requirements of physics in general and quantum physics in particular do call for the smallest possible uncertainties. The problem is brought into light starkly by asking the following question [4]: 'In the 21st century, why should a piece of Pt–Ir alloy forged in the 19th century that sits in a vault in Sèvres restrict our knowledge of the values of h and m_e?'

The fourth assumption is that a new definition of a unit should not introduce a discontinuity in the value of the unit. This means that the chosen values of the constants h, e, k and N_A used in the new definitions should be as close to their SI values as current knowledge allows. The implication is that one could not choose, for example, the values of the Planck constant and elementary charge, h_{90} and e_{90}, implied by the conventional values of the Josephson and von Klitzing constants, $K_{J-90} = 2e_{90}/h_{90} = 483\,594.9\,\mathrm{GHz\,V^{-1}}$ and $R_{K-90} = h_{90}/e_{90}^2 = 25\,812.807\,\Omega$ [11], since K_{J-90} and R_{K-90} are known from the CODATA 2002 set of recommended values of the constants to deviate non-negligibly from the best values of the Josephson and von Klitzing constants K_J and R_K expressed in SI units [9] (also see the last paragraph of section 5.2).

1.3. Requirements regarding data expected by the end of 2010

The 2002 CODATA recommended values of h, e, k and N_A are [9]

$$\begin{aligned} h &= 6.626\,0693(11)\times10^{-34}\,\mathrm{J\,s}\quad[1.7\times10^{-7}],\\ e &= 1.602\,176\,53(14)\times10^{-19}\,\mathrm{C}\quad[8.5\times10^{-8}],\\ k &= 1.380\,6505(24)\times10^{-23}\,\mathrm{J\,K^{-1}}\quad[1.8\times10^{-6}],\\ N_A &= 6.022\,1415(10)\times10^{23}\,\mathrm{mol^{-1}}\quad[1.7\times10^{-7}], \end{aligned}\tag{1}$$

where as usual the number in parentheses is the numerical value of the standard uncertainty referred to the last two digits of the quoted value and the number in square brackets is the corresponding relative standard uncertainty u_r. In the 2002 adjustment, h and the molar gas constant R are adjusted constants (that is, are taken as variables or 'unknowns' of the adjustment) and hence are directly determined by the adjustment itself; e, k and N_A are then calculated from the relations

$$e = \left(\frac{2\alpha h}{\mu_0 c_0}\right)^{1/2}\qquad k = \frac{R}{N_A}\qquad N_A = \frac{c_0 A_r(e) M_u \alpha^2}{2R_\infty h},\tag{2}$$

where $\mu_0 = 4\pi\times10^{-7}\,\mathrm{N\,A^{-2}}$ is the magnetic constant, $M_u = 10^{-3}\,\mathrm{kg\,mol^{-1}}$ is the molar mass constant, and α, $A_r(e)$ and R_∞, which are the fine-structure constant, relative atomic mass of the electron and the Rydberg constant, respectively, are also adjusted constants. The respective uncertainties of the 2002 recommended values of R and these three constants are $u_r(R) = 1.7 \times 10^{-6}$, $u_r(\alpha) = 3.3 \times 10^{-9}$, $u_r[A_r(e)] = 4.4 \times 10^{-10}$ and $u_r(R_\infty) = 6.6 \times 10^{-12}$. Thus, the uncertainty of h plays the dominant role by far in determining the uncertainty of e and N_A, while the uncertainty of R plays a similar role in determining the uncertainty of k.

The 2002 recommended value of R is essentially the weighted mean of two independent results for the speed of sound in argon obtained at a temperature close to and known in terms of the triple point of water T_{TPW}, one from the National Institute of Standards and Technology (NIST), USA, with $u_r = 1.8 \times 10^{-6}$, and the other from the National Physical Laboratory (NPL), UK, with $u_r = 8.4 \times 10^{-6}$ [9]. Although the two results are consistent, because of the large difference in their uncertainties, the 2002 recommended value of R, and hence the 2002 recommended value of the Boltzmann constant k with $u_r(k) = 1.8 \times 10^{-6}$, is to a very large extent determined by the NIST result. The important point to note here is that if the 2002 CODATA recommended value of k were taken to be exact and used to define the kelvin, its uncertainty would be transferred to the value of T_{TPW}. This means that if such a new definition were to be adopted today, our best estimate of the value of T_{TPW} would still be 273.16 K, but instead of this value being exact as a result of the definition of the kelvin as is now the case, the uncertainty associated with the estimate would become $u_r(T_{TPW}) = 1.8 \times 10^{-6}$, which corresponds to 0.49 mK. The issue that the thermometry community must address is whether or not this uncertainty is acceptable and, if not, how small an uncertainty is required. For our purposes here, we shall assume that the experiments currently underway to measure R or k [12–14] will achieve by the end of 2010 a relative standard uncertainty about a factor of two smaller than the current u_r of approximately 2×10^{-6}, so that $u_r(T_{TPW})$ will be reduced to about 1×10^{-6}, corresponding to about 0.25 mK, and that this will be small enough for the redefinition to proceed in 2011. In this connection, it is worth noting that u_r of the other defining fixed points of the International Temperature Scale of 1990, ITS-90, which is the basis for all practical thermometry, are significantly larger (see section 4.1.3).

Unfortunately, the situation regarding the Planck constant is more complex. The 2002 CODATA recommended value is based mainly on five input data [9], four of which are electrical and one from x-ray-crystal-density (XRCD) experiments using silicon. In brief, these are (i) a NIST moving-coil watt-balance result for h, which is the dominant contributor to the recommended value, (ii) a similar NPL result for h, (iii) a combined National Metrology Institute of Japan (NMIJ), Physikalisch-Technische Bundesanstalt (PTB), Germany, and Institute for Reference Materials and Measurements (IRMM), Belgium, result for the molar volume of silicon $V_m(\mathrm{Si})$ obtained by the XRCD method, (iv) a National Measurement Laboratory (NML), Australia, mercury electrometer result for the Josephson constant K_J and (v) a PTB capacitor voltage-balance result for K_J. A watt balance determines h directly, while for measurements of K_J and $V_m(\mathrm{Si})$, h is obtained from the relations

$$h = \frac{8\alpha}{\mu_0 c_0 K_J^2} \qquad h = \frac{\sqrt{2} c_0 A_r(e) M_u \alpha^2 d_{220}^3}{R_\infty V_m(\mathrm{Si})}, \tag{3}$$

where d_{220} is the $\{220\}$ lattice spacing of an ideal crystal of naturally occurring silicon. Since u_r for the NML and PTB values of K_J are 2.7×10^{-7} and 3.1×10^{-7}, respectively, and, as noted above, $u_r(\alpha) = 3.3 \times 10^{-9}$, the uncertainties of the two values of h deduced from the two measurements of K_J are completely dominated by the uncertainties of K_J. An analogous but less definitive statement applies to the uncertainty of the value of h deduced from $V_m(\mathrm{Si})$, since its 3.0×10^{-7} relative standard uncertainty, while much larger than $2u_r(\alpha)$, $u_r[A_r(e)]$ and $u_r(R_\infty)$ as given above, is less than three times larger than $3u_r(d_{220}) = 1.1 \times 10^{-7}$. The values of u_r for the five values of h are 8.7×10^{-8}, 2.0×10^{-7}, 3.2×10^{-7}, 5.4×10^{-7} and 6.3×10^{-7}, respectively.

The two watt-balance values of h agree well as do the two K_J values, and all four agree well among themselves, but the value of h deduced from $V_m(\mathrm{Si})$ is in significant disagreement with these four values; it exceeds the weighted mean of the latter by the fractional amount $1.12(33) \times 10^{-6}$, where the uncertainty is the standard uncertainty of the difference. Further, in a least-squares adjustment involving all the data considered for possible inclusion in the 2002 CODATA adjustment, the normalized residual of the input datum $V_m(\mathrm{Si})$ is -3.18. (The normalized residual of a given input datum $r_i = (q_i - \hat{q}_i)/u(q_i)$ is the ratio of the difference between the value of the input datum q_i and the best estimated value $\hat{q}_i$ of that datum resulting from the adjustment to the *a priori* uncertainty $u(q_i)$ assigned to the datum.) To deal with this inconsistency, the CODATA Task Group decided to weight the *a priori* assigned uncertainties of all five data by the multiplicative factor 2.325 in the final adjustment on which the 2002 CODATA recommended values are based, in order to reduce $|r_i|$ from 3.18 to the acceptable value 1.50. The end result of this decision is an increase in u_r of the 2002 recommended value of h and of those constants that depend strongly on h, such as e and N_A, by a factor of about 2.3 compared with the value of u_r that would have resulted without the weighting factor 2.325.

The discrepancy just described was a major point of concern at the February 2005 Royal Society Discussion Meeting and the March to July 2005 meetings of the CCs and the CODATA Task Group discussed in section 1.1. As a consequence, we now accept the consensus view that before the new definitions are adopted the available data should be such that one can be confident in the CODATA recommended value of h. This is generally taken to mean that the existing discrepancy between the electrical and XRCD results is satisfactorily resolved. Further, the CCM in its Recommendation G 1 (2005) to the CIPM recommends that u_r 'of the best realization of the definition of the kilogram does not exceed two parts in 10^8, at the level of one kilogram', which implies a final $u_r(h)$ of about 2×10^{-8}. Thus, we assume in this paper that the several watt-balance experiments currently underway (see, for example, the review [15]), and the international effort to determine d_{220} and $V_m(\mathrm{Si})$ by the XRCD method using a highly enriched silicon sample with an amount of substance fraction of $^{28}\mathrm{Si}$ of 99.985% [16], will provide data by 31 December 2010, the closing date of the 2010 CODATA adjustment, that lead to such a value of h. It is worth noting that a significant step in this direction has been taken by the NIST watt-balance group—they recently reported a value of

Table 1. The definitions of the kilogram, ampere, kelvin and mole discussed in sections 2.2 and 2.3 that link these units to exact values of the Planck constant h, elementary charge e, Boltzmann constant k and Avogadro constant N_A, respectively.

Kilogram	Ampere	Kelvin	Mole
(kg-1a) The kilogram is the mass of a body whose equivalent energy is equal to that of a number of photons whose frequencies sum to exactly $[(299\,792\,458)^2/662\,606\,93] \times 10^{41}$ hertz. (kg-1b) The kilogram is the mass of a body whose de Broglie–Compton frequency is equal to exactly $[(299\,792\,458)^2/(6.626\,069\,3 \times 10^{-34})]$ hertz.	(A-1) The ampere is the electric current in the direction of the flow of exactly $1/(1.602\,176\,53 \times 10^{-19})$ elementary charges per second.	(K-1) The kelvin is the change of thermodynamic temperature that results in a change of thermal energy kT by exactly $1.380\,650\,5 \times 10^{-23}$ joule.	(mol-1) The mole is the amount of substance of a system that contains exactly $6.022\,141\,5 \times 10^{23}$ specified elementary entities, which may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.
(kg-2) The kilogram, unit of mass, is such that the Planck constant is exactly $6.626\,069\,3 \times 10^{-34}$ joule second.	(A-2) The ampere, unit of electric current, is such that the elementary charge is exactly $1.602\,176\,53 \times 10^{-19}$ coulomb.	(K-2) The kelvin, unit of thermodynamic temperature, is such that the Boltzmann constant is exactly $1.380\,650\,5 \times 10^{-23}$ joule per kelvin.	(mol-2) The mole, unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles, is such that the Avogadro constant is exactly $6.022\,141\,5 \times 10^{23}$ per mole.

h with $u_r = 5.2 \times 10^{-8}$ that is consistent with the previous NIST result but was obtained using an almost completely new apparatus [17].

We conclude this section by noting that ongoing work promises to lead in the near future to a value of the fine-structure constant α with u_r less than 10^{-9} [18, 19], which is to be compared with the uncertainty of the CODATA 2002 recommended value, $u_r(\alpha) = 3.3 \times 10^{-9}$ [9]. We therefore assume that the 2010 CODATA recommended value will have an uncertainty $u_r(\alpha)$ slightly smaller than 10^{-9}, which represents a reduction in uncertainty of somewhat larger than a factor of three.

2. Unit definitions

2.1. Introduction

In this section we propose possible wordings for new definitions of the kilogram, ampere, kelvin and mole that link these units to exact values of h, e, k and N_A, respectively, and, to stimulate broader thinking about the SI, a possible way to redefine the entire International System explicitly in terms of fundamental constants without associating a particular unit with a particular constant. The final wordings of any new definitions will eventually be chosen by the CIPM for submission to the CGPM for the latter's adoption. However, the choice does not have to be made by the CIPM until its 99th meeting in 2010 if the new definitions are to be approved by the 24th CGPM in 2011, so there is adequate time for a full discussion of the various alternatives.

The possible alternatives that we suggest for each unit are presented in separate sections below and are summarized in table 1 for easy reference. In the table, each definition in the first row explicitly defines a unit in terms of a particular quantity of the same kind as the unit and, through a simple relationship implied by the definition itself or one or more laws of physics, implicitly fixes the value of a fundamental constant; we call these 'explicit-unit definitions'. Each definition in the second row explicitly fixes the value of a fundamental constant and, through a simple relationship implied by the definition itself or one or more laws of physics, implicitly defines a unit; we call these 'explicit-constant definitions'. It should be understood, however, that the alternative definitions for the same unit are in fact equivalent; they are only different ways of stating the same definition, and in no way should the choice of words be regarded as final at this stage. For ease of identification, a '1' in a unit identifier, such as (kg-1a), indicates an explicit-unit definition, while a '2' in the identifier, such as (kg-2), indicates an explicit-constant definition.

All the current definitions of the SI base units can be interpreted as being of the explicit-unit type, although the 'constant' implicitly fixed by each definition is not necessarily a traditional fundamental physical constant. For example, the current SI definition of the second fixes the value of $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$, the ground state hyperfine splitting transition frequency of the caesium 133 atom, and the definition of the kelvin fixes the value of T_{TPW}, the thermodynamic temperature of the triple point of water. The constants whose values are fixed by the definitions of the five other SI base units—the metre, kilogram, ampere, mole and candela—are c_0, $m(\mathcal{K})$, μ_0, the molar mass of carbon 12, $M(^{12}\mathrm{C})$, and the spectral luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, $K(\lambda_{555})$ (the wavelength of radiation of frequency 540×10^{12} hertz is approximately 555 nanometres). One problem that such definitions have, including those in row one of table 1, and which explicit-constant definitions such as those in row two avoid, is that the constant to which the unit is linked and its value are not readily apparent. However, the effect in either case is both to define a unit and to fix the value of a constant.

For specificity, here and throughout the paper we use the 2002 CODATA set of recommended values of the constants as the basis for all stated numerical values. But it must be recognized that the number of digits given in the 2002 set for the constants required in the new definitions is not adequate to prevent the introduction of significant discontinuities in the magnitudes of the redefined units or to avoid significant rounding errors in the values of other fundamental constants calculated from those in the definitions. The question of the proper number of digits to be used will, of course, be sorted out at the time the final values of the required constants are selected, and it can be safely assumed that the values will be chosen to preserve continuity in the magnitudes of the units. Further, as intimated in section 1.3, it is expected that these values will be based on the 2010 CODATA set of recommended values. (The uncertainties that we use in this paper for various constants, when needed, are also given in section 1.3.)

2.2. *Explicit-unit definitions*

There are obviously many possible wordings for each explicit-unit definition, but for simplicity we give only two for the kilogram and one each for the other three units under consideration.

2.2.1. Kilogram. Two different approaches to the formulation of an explicit-unit definition of the kilogram have been proposed, but they lead to very similar definitions. One is based on equating the equivalent energy of a body of mass one kilogram to the energy of a number of photons [7, 11, 20]. The other is based on assigning a specific value to the 'Compton frequency' (occasionally called the 'de Broglie–Compton frequency') of a body of mass one kilogram [13, 14]. One form of the first approach might read as follows.

> (kg-1a) The kilogram is the mass of a body whose equivalent energy is equal to that of a number of photons whose frequencies sum to exactly $[(299\,792\,458)^2/662\,606\,93] \times 10^{41}$ hertz.

The value of h to which this definition links the kilogram follows from the Einstein relation $E = mc_0^2$ and the relation $E = h\nu = hc_0/\lambda$ for photons of frequency ν or wavelength λ. Thus $h = (1\,\mathrm{kg})(299\,792\,458\,\mathrm{m\,s^{-1}})^2/\{[(299\,792\,458)^2/662\,606\,93] \times 10^{41}\,\mathrm{Hz}\}$ or $h = 6.626\,069\,3 \times 10^{-34}\,\mathrm{J\,s}$, where, of course, the joule, $\mathrm{J} = \mathrm{m^2\,kg\,s^{-2}}$, is the SI unit of energy.

A version of the definition of the second type, proposed by the working group of the *Académie des Sciences* and submitted to the CCU for its 17th meeting in 2005, reads as follows:

> (kg-1b) The kilogram is the mass of a body whose de Broglie–Compton frequency is equal to exactly $[(299\,792\,458)^2/(6.626\,069\,3 \times 10^{-34})]$ hertz,

where we have repositioned the word 'exactly' for consistency with the other definitions. In this proposal (also see [13, 14]), the de Broglie–Compton frequency of a body of mass m is defined according to $\nu_m = c_0/\lambda_{\mathrm{C},m} = (mc_0^2)/h$, where $\lambda_{\mathrm{C},m} = h/(mc_0)$ is referred to as the Compton wavelength of the body, analogous to the Compton wavelength of the electron $\lambda_{\mathrm{C}} = h/(m_{\mathrm{e}}c_0)$. The value of ν_m specified in definition (kg-1b) implies that the value of h is exactly known because $h = mc_0^2/\nu_m = 1\,\mathrm{kg} \times (299\,792\,458\,\mathrm{m\,s^{-1}})^2/\{[(299\,792\,458)^2/(6.626\,069\,3 \times 10^{-34})]\,\mathrm{Hz}\}$, which yields $h = 6.626\,069\,3 \times 10^{-34}\,\mathrm{J\,s}$.

Both definitions (kg-1a) and (kg-1b) have their own special merits. The first has the advantage of being based on relatively well-known fundamental relations and thus is likely to be recognized by a comparatively large audience. The second has the advantage of being based on a property of a body (although rather unphysical), that is, its de Broglie–Compton frequency, and, as pointed out in [13, 14], the ratio h/m is the quantity that is usually observed and measured in the real world. For example, atom interferometry measures $h/m({}^{A}X)$ where ${}^{A}X$ is an atom such as ${}^{133}\mathrm{Cs}$, and a watt balance measures h/m_{s}, where m_{s} is a macroscopic standard of mass, typically between 100 g and 1 kg, used in the 'weighing' portion of the experiment. On the other hand, it should be recognized that for a mass of 1 kg, ν_m is an unrealistically large frequency; its corresponding wavelength, $\lambda_{\mathrm{C},1\,\mathrm{kg}} = h/(1\,\mathrm{kg}\;c_0) \approx 2.2 \times 10^{-42}\,\mathrm{m}$, is orders of magnitude smaller than the Planck length $l_{\mathrm{P}} \approx 1.6 \times 10^{-35}\,\mathrm{m}$ [9].

2.2.2. Ampere. A rather straightforward explicit-unit definition that links the ampere to an exact value of e is as follows:

> (A-1) The ampere is the electric current in the direction of the flow of exactly $1/(1.602\,176\,53 \times 10^{-19})$ elementary charges per second.

Identification of the value of e follows from the relation $It = Ne$, where I is current, t is time interval and N is number of elementary charges. Taking $I = 1\,\mathrm{A}$, $t = 1\,\mathrm{s}$ and $N = 1/(1.602\,176\,53 \times 10^{-19})$ as stated in the definition, we obtain from this simple relation $e = (1/N)\,\mathrm{A\,s} = 1.602\,176\,53 \times 10^{-19}\,\mathrm{C}$, since the coulomb, C, is the special name for the ampere second, A s.

This definition specifies that the direction of the current is in the direction of the flow of positive charges, since the elementary charge is defined to be a positive quantity (that is, the absolute value of the charge of the electron or the charge of the proton).

2.2.3. Kelvin. The kelvin, unit of thermodynamic temperature, can be linked to an exact value of k by a definition that explicitly specifies the value of the conversion factor between it and the joule, since the conversion factor is in fact the Boltzmann constant k, unit $\mathrm{J\,K^{-1}}$. A definition proposed in a recent review [12] that is of the explicit-unit type accomplishes this by simply specifying the energy corresponding to a temperature interval of one kelvin:

> (K-1) The kelvin is the change of thermodynamic temperature that results in a change of thermal energy kT by exactly $1.380\,650\,5 \times 10^{-23}$ joule,

where we have inserted the word ‘exactly’ for consistency with the other definitions. Since this definition states that $k \times (1\,\mathrm{K}) = 1.380\,650\,5 \times 10^{-23}\,\mathrm{J}$, it clearly has the effect of fixing the value of the Boltzmann constant to be $k = 1.380\,650\,5 \times 10^{-23}\,\mathrm{J\,K^{-1}}$.

2.2.4. Mole. Before giving the definition of the mole, it is useful to review the implications of the current definition of the mole, which assumes that the kilogram is independently defined. It reads [1] as follows:

1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12; its symbol is ‘mol.’
2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.

In this definition, it is understood that unbound atoms of carbon 12, at rest in their ground state, are referred to. (When the definition of the mole is quoted, it is conventional also to include this remark, which was approved by the 69th CIPM in 1980.)

An important consequence of this definition is that one mole of any specified entity X contains the exact same number of entities. The Avogadro constant N_A is defined as this exact number of entities per mole, and its approximate value is $6.022 \times 10^{23}\,\mathrm{mol^{-1}}$.

The molar mass $M(X)$ of a specified entity X is the mass of one mole of X, and thus it follows from the definitions of molar mass and N_A that for any entity X

$$M(X) = N_\mathrm{A} m(X), \tag{4}$$

where $m(X)$ is the mass of entity X. Since as a direct consequence of the current definition of the mole $M(^{12}\mathrm{C}) = 0.012\,\mathrm{kg\,mol^{-1}}$, equation (4) implies $N_\mathrm{A} m(^{12}\mathrm{C}) = 0.012\,\mathrm{kg\,mol^{-1}}$, or in a more compact notation, $M(^{12}\mathrm{C}) = N_\mathrm{A} m(^{12}\mathrm{C}) = 12 M_\mathrm{u}$, where, as previously indicated, $M_\mathrm{u} = 10^{-3}\,\mathrm{kg\,mol^{-1}}$ is the molar mass constant. This implies that if one tries to adopt an exact value for N_A, $m(^{12}\mathrm{C})$ in the unit kilogram would also have an exactly known value so that $1\,\mathrm{kg} = [N_\mathrm{A}\,\mathrm{mol}\,m(^{12}\mathrm{C})]/0.012$, in contradiction with our starting assumption that the kilogram is independently defined. Thus, only if one defines the mole so that it is linked to an exact value of N_A in a way that is independent of the kilogram can one avoid such an inconsistency in definitions.

An explicit-unit definition that accomplishes this goal reads as follows.

> (mol-1) The mole is the amount of substance of a system that contains exactly $6.022\,141\,5 \times 10^{23}$ specified elementary entities, which may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.

That it does so can be seen as follows: since the Avogadro constant N_A is defined as the number of entities per mole, definition (mol-1) implies that $1\,\mathrm{mol} = (6.022\,141\,5 \times 10^{23})/N_\mathrm{A}$, or $N_\mathrm{A} = 6.022\,141\,5 \times 10^{23}\,\mathrm{mol^{-1}}$. Clearly, there is no need for the CIPM-approved remark included by convention with the current definition of the mole because neither this definition nor its counterpart explicit-constant definition (mol-2) (the latter given below and in table 1) is based on a given mass of carbon 12 atoms.

Definitions (mol-1) and (mol-2) not only retain the basic definition of the Avogadro constant as the number of entities per mole and link the mole to the exact value of the Avogadro constant $N_\mathrm{A} = 6.022\,141\,5 \times 10^{23}\,\mathrm{mol^{-1}}$ without placing any restrictions on the kilogram, they also (i) retain the basic relationship between the molar mass of an entity X and the mass of the entity as given by equation (4), and (ii) are more readily understood because of their simplicity than is the current definition. Indeed, definition (mol-1) has the additional advantage of making eminently clear that the mole is a measure of a number of specified entities and has nothing to do with mass. Further, as shown in section 4.1.4, all the following can remain unchanged: the current definition of the molar mass constant, $M_\mathrm{u} = 10^{-3}\,\mathrm{kg\,mol^{-1}}$, the current definition of the unified atomic mass unit u (also called the dalton, Da) and atomic mass constant m_u, $1\,\mathrm{u} = m_\mathrm{u} = m(^{12}\mathrm{C})/12$, and the current definition of the quantity relative atomic mass, $A_\mathrm{r}(X) = m(X)/m_\mathrm{u}$, which implies $A_\mathrm{r}(^{12}\mathrm{C}) = 12$ as at present and that existing compilations of relative atomic masses need not be revised. In fact, the only change that would need to be dealt with is the replacement of the usual expression relating molar mass to relative atomic mass, $M(X) = A_\mathrm{r}(X) M_\mathrm{u}$, by the expression $M(X) = (1+\kappa) A_\mathrm{r}(X) M_\mathrm{u}$, where the factor $(1+\kappa)$ is a correction factor to allow for any small deviation of the molar mass from the value $A_\mathrm{r}(X) M_\mathrm{u}$. However, as shown in section 4.1.4, the deviation of this ‘molar mass factor’ from unity as well as its uncertainty is never likely to be greater than a few parts in 10^9. Hence for all practical purposes molar mass may still be calculated from the product $A_\mathrm{r}(X) M_\mathrm{u}$ and $M(^{12}\mathrm{C})$ may still be taken to be equal to $12\,\mathrm{g\,mol^{-1}}$.

2.3. Explicit-constant definitions

Such a definition simply states that the unit is defined by assigning to a particular fundamental constant an exact, specified value. The explicit-constant definition that we propose, applicable to all base units, is of the general form ‘The [name of base unit], unit of the [name of base quantity], is such that the [name of fundamental constant] is exactly [value of fundamental constant].’ For example, if the current definition of the metre, which links this unit to the exact value of the speed of light in vacuum $c_0 = 299\,792\,458\,\mathrm{m\,s^{-1}}$, was to be worded in this way, it would read as follows.

> (m-2) The metre, unit of length, is such that the speed of light in vacuum is exactly 299 792 458 metres per second.

Such a definition has the advantages that it is simple, concise and makes clear the fundamental constant to which the unit is linked and the exact value of that constant. If this general form were chosen, it would be appropriate to choose definitions of the same form for all seven base units. Thus, for the second and candela we would have

> (s-2) The second, unit of time, is such that the ground state hyperfine splitting transition frequency of the caesium 133 atom is exactly 9 192 631 770 hertz.

> (cd-2) The candela, unit of luminous intensity in a given direction, is such that the spectral luminous efficacy of monochromatic radiation of frequency 540×10^{12} hertz is exactly 683 lumens per watt.

It follows from the template given above that the explicit-constant definitions for the kilogram, ampere, kelvin and mole that link these units to exact values of h, e, k and N_A would read as follows:

> (kg-2) The kilogram, unit of mass, is such that the Planck constant is exactly $6.626\,069\,3 \times 10^{-34}$ joule second.
>
> (A-2) The ampere, unit of electric current, is such that the elementary charge is exactly $1.602\,176\,53 \times 10^{-19}$ coulomb.
>
> (K-2) The kelvin, unit of thermodynamic temperature, is such that the Boltzmann constant is exactly $1.380\,650\,5 \times 10^{-23}$ joule per kelvin.
>
> (mol-2) The mole, unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles, is such that the Avogadro constant is exactly $6.022\,141\,5 \times 10^{23}$ per mole.

These definitions obviously represent a significant departure from the current explicit-unit definitions of SI base units as well as from the proposed explicit-unit definitions in row one of table 1. It is therefore useful to show how adopting an exact value for $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$, c_0, h, e, k, N_A and $K(\lambda_{555})$ defines the second, metre, kilogram, ampere, kelvin, mole and candela, respectively, and provides a means of realizing these units in the laboratory. We do so by making use of the quantity calculus, where quantities, units and numbers are all treated by the ordinary rules of algebra. Thus, from definition (s-2) for the second we have

$$\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs} = 9\,192\,631\,770\,\mathrm{Hz},$$

which, recalling that $\mathrm{Hz} = \mathrm{s}^{-1}$, leads to

$$1\,\mathrm{s} = 9\,192\,631\,770/\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}.$$

Since $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$ is a real frequency characteristic of the ^{133}Cs atom and 9 192 631 770 is simply a number, this relation shows how the second is completely specified and capable of being realized in the laboratory by adopting an exact value for $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$. The other units are similar. From explicit-constant definition (m-2) for the metre we have

$$c_0 = 299\,792\,458\,\mathrm{m\,s^{-1}},$$

which means that

$$1\,\mathrm{m} = (c_0\,\mathrm{s})/299\,792\,458.$$

Again, since c_0 is a real speed, s can be realized from $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$ and 299 792 458 is simply a number, the metre is completely specified and realizable in the laboratory by adopting an exact value for c_0.

For the kilogram, noting that the joule, J, is simply a special name and symbol for $\mathrm{m^2\,kg\,s^{-2}}$, explicit-constant definition (kg-2) states

$$h = 6.626\,069\,3 \times 10^{-34}\,\mathrm{m^2\,kg\,s^{-2}\,s},$$

from which it follows that

$$1\,\mathrm{kg} = (h\,\mathrm{m^{-2}\,s})/(6.626\,069\,3 \times 10^{-34}).$$

Thus, since the mass of a body can be related to h in the laboratory by, for example, a watt balance, m and s can also be realized in the laboratory and $6.626\,069\,3 \times 10^{-34}$ is simply a number, adopting an exact value for h completely specifies the kilogram and allows it to be realized for practical use.

Because the other four units follow the same pattern, we simply give the final equation for each with a minimum of comment, noting only that a current can be related to $e\,\mathrm{s}^{-1}$ by means of the Josephson and quantum Hall effects, a thermodynamic temperature can be related to k by several different experiments, the molar mass of an entity can be related to N_A through its relative atomic mass and the known values of other constants (see section 4.1.4) and a luminous intensity can be related to $K(\lambda_{555})$ using a cryogenic radiometer.

$$1\,\mathrm{A} = e\,\mathrm{s}^{-1}/(1.602\,176\,53 \times 10^{-19})$$

$$1\,\mathrm{K} = (1.380\,650\,5 \times 10^{-23})/(k\,\mathrm{m^{-2}\,kg^{-1}\,s^{2}})$$

$$1\,\mathrm{mol} = (6.022\,141\,5 \times 10^{23})/N_\mathrm{A}$$

$$1\,\mathrm{cd} = [K(\lambda_{555})\,\mathrm{m^2\,kg\,s^{-3}\,sr^{-1}}]/683.$$

2.4. *Could the units of the SI be defined simply in terms of fixed values of a set of constants without associating a particular unit with a particular constant?*

In this section we present for further discussion the broad outline of a novel proposal that seems to us to be a logical extension of what has just been presented in the previous section. The full set of explicit-constant definitions suggests that one could go a step further and simply state that the units of the SI are completely specified by fixing the values of a particular set of seven fundamental constants (broadly interpreted in the case of $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$ and $K(\lambda_{555})$). This version of the SI would be implemented through a CGPM Resolution that would define the SI to be the system of units in which the above seven constants have specified, exact values when expressed in those units. Further, the resolution would abrogate each of the current definitions of the base units. This means, for example, that the current definition of the mole based on the kilogram would no longer apply, so that fixing the Avogadro constant would only define the mole with no consequence for the kilogram and that the mass of the international prototype would no longer serve as the basis for the definition of the kilogram. Thus we could say the following:

The International System of Units, the SI, is the system of units scaled so that the

(1) ground state hyperfine splitting transition frequency of the caesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_\mathrm{hfs}$ is 9 192 631 770 hertz,

(2) speed of light in vacuum c_0 is 299 792 458 metres per second,
(3) Planck constant h is $6.626\,069\,3 \times 10^{-34}$ joule second,
(4) elementary charge e is $1.602\,176\,53 \times 10^{-19}$ coulomb,
(5) Boltzmann constant k is $1.380\,650\,5 \times 10^{-23}$ joules per kelvin,
(6) Avogadro constant N_A is $6.022\,141\,5 \times 10^{23}$ per mole and
(7) spectral luminous efficacy of monochromatic radiation of frequency 540×10^{12} hertz $K(\lambda_{555})$ is 683 lumens per watt.

Accompanying this definition of the SI would be a list of representative units, together with a representative list of the quantities whose values could be expressed in those units. This list would include, of course, the metre for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for thermodynamic temperature, the mole for amount of substance and the candela for luminous intensity, as well as the current 22 SI derived units with special names and symbols such as the radian, newton, volt, lumen and katal and some of their corresponding quantities [1]. Such a list could in fact be taken from, for example, tables 1–4 in [1], the BIPM SI Brochure. This single definition and list, together with the same system of quantities and laws of physics upon which the present SI rests, establishes the entire system without the introduction of base units and derived units—all units are on an equal footing. Further, there is no need to be concerned about whether or not adopting exact values for these seven constants fully specifies the SI, for we know that these constants define the seven SI base units and that the SI as presently constructed is fully specified by those units[3]. This version of the SI is only a mild departure from the guiding assumption discussed in the first paragraph of section 1.2, inasmuch as the quantities and units on which it is based are the same as the current SI; the only difference is that the categorization of units as ‘base’ or ‘derived’ is no longer applicable and this we see as a logical extension of current thinking.

The practical realization of any unit of this new version of the SI, whether it is one of the present base or derived units or not, would be by employing a method (a primary method) defined by an appropriate equation of physics linking the unit in question to one or more of the fixed constants. For example, the volt and ohm would be realized through the equations of the Josephson and quantum Hall effects using the exact values of h and e; the kelvin through a primary thermometer using the exact values of k or R, and so on. The user would be at liberty to use whichever equation of physics and method is considered most appropriate. The CIPM could decide, however, to formalize some of these methods as a *mise-en-pratique*.

Looking further to the future, it is of interest to speculate about eventually replacing the definition of the second based on $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ with a definition that links the second to an exact value of the familiar and highly important Rydberg constant R_∞. In this case, entry (1) in the numbered list above, including the three words that precede it, would read ‘so that the (1) Rydberg constant R_∞ is 10 973 731.568 525 inverse metres’. At present, the theory and experimental determination of hydrogen and deuterium transition frequencies are not sufficiently accurate to do this, but they could be in the future [21, 22]. In the formulation of the SI considered here, such a replacement could simply be made with no other change. The fact that the Rydberg constant has the unit of inverse metre and would replace a constant that has the unit of inverse second would not matter; the product $c_0 R_\infty$ would be an exactly known frequency that could be related by theory to an accurately measurable transition frequency in hydrogen.

A major advantage of the proposed new approach is that it does away entirely with the need to specify base units and derived units and hence the confusion that this requirement has long been recognized to engender, not the least of which is the arbitrariness of the distinction between base units and derived units. This need is eliminated by no longer having a unique, one-to-one correspondence between a particular unit and a particular fundamental constant. It thus does away with a situation such as that which exists with the explicit-constant definition for the ampere, (A-2) in section 2.3, in which the unit of current is defined in terms of a constant, the elementary charge, the unit of which is not the ampere but the ampere second, or coulomb. Such cross-referencing between units in definitions can be avoided by not linking particular constants to particular units.

We emphasize that no matter which direction the CIPM chooses to take—the explicit-unit approach, the explicit-constant approach or this last approach that defines the entire SI without linking a particular unit to the exact value of a particular constant—the same measurement system will result. In practice, if not formally, the base units and derived units will be indistinguishable and the seven constants listed above, that is, $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$, c_0, h, e, k, N_A and $K(\lambda_{555})$, will form the basis of the system.

3. Considerations in redefining the kilogram, ampere, kelvin and mole

The redefinition of four SI base units as discussed in this paper will lead to changes in the SI that are quite profound. It will have important benefits for both metrology and science, bringing us a step closer to the long-sought but elusive goal of having every SI base unit linked to an important fundamental constant of nature and having a large number of the fundamental constants either exactly known or known with very small uncertainties when expressed in terms of SI units. Nevertheless, because the changes are potentially of such wide-ranging impact, before being made they need to be discussed broadly by both the metrological and scientific communities. To foster and provide a focus for such discussion, we suggest in this section points to be considered and actions to be taken in the redefinition process (also see sections 4 and 5).

3.1. Timing of the implementation of the new definitions

We suggest that all four of the proposed unit redefinitions—kilogram, ampere, kelvin and mole—should be addressed in a single resolution. It is our view that they form a coherent, self-consistent set that should not be broken apart and that only by

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[3] However, the choice of constants used to define the SI is not unique. For example, statements (3) and (4), which fix the values of h and e, could be replaced by statements that fix the values of the Josephson and von Klitzing constants $K_J = 2e/h$ and $R_K = h/e^2$. If this were to be done, statements (5) and (6) could then be replaced by statements that fix the Stefan–Boltzmann constant $\sigma = (2/15)\pi^5 k^4/(h^3 c_0^2)$ and Faraday constant $F = N_A e$.

adopting all four new definitions at the same time (assuming the data relevant to determining h and k are satisfactory) will the maximum benefits accrue to both metrology and the fundamental constants. For example, if all four definitions are put into effect simultaneously,

(i) one can take advantage of the fact that a large segment of the measurement community will undoubtedly be focused on the changes, thus minimizing any associated inconvenience to practical metrology,
(ii) the maximum number of fundamental constants become exactly known, including all factors required to convert the value of an energy-related quantity expressed in either joules, kilograms, inverse metres, hertz, kelvins or electronvolts to a value expressed in one of the other units without any additional uncertainty,
(iii) in particular, the definitions of the kilogram and ampere that link these units to exact values of h and e should be viewed as an indivisible pair because both are required for the Josephson constant $K_\mathrm{J} = 2e/h$ and von Klitzing constant $R_\mathrm{K} = h/e^2$ to become exactly known, thereby allowing the Josephson and quantum Hall effects to be used to realize the SI ampere, volt, ohm, watt, farad and henry with no uncertainty contribution from the uncertainty of K_J and/or R_K, and
(iv) the value of the SI as a common measurement language for intelligible communication between practical metrology and quantum physics will be significantly improved.

This last point is critical and worthy of elaboration. It should be recognized that the needs of these two communities are different and hence that a measurement system that is designed solely to meet those of one community will inadequately meet those of the other. This implies that any system designed to meet the requirements of both in such a way that the two communities can readily communicate must, by necessity, be a compromise system. Although to date the SI has served as such a system reasonably well, we believe that recent advances in physics, both theoretical and experimental, have pushed the current SI to the point where, unless the new definitions are adopted in 2011, it may soon become an impediment to future progress, most especially to the many efforts currently underway to improve our knowledge of the values of the fundamental physical constants expressed in SI units.

3.2. Key points

It would be useful when introducing the new definitions to provide some of the background and reasons for such a significant modification to the SI. For example, in addition to the points made in items (ii) and (iii) above regarding the benefits of energy conversion factors as well as K_J and R_K being exactly known, the following additional points should be covered:

(i) recognition of the significant efforts expended by the national metrology institutes (NMIs) as well as the BIPM over the last several decades to advance the SI by relating SI base units to the invariants of nature, the fundamental physical constants, with the metre and the speed of light in vacuum being a prime example,
(ii) that the kilogram is the only SI base unit still defined in terms of a material artefact, the international prototype of the kilogram, that the definitions of the ampere, mole and candela depend on the kilogram and that the mass of the international prototype $m(\mathcal{K})$ is undoubtedly changing with time,
(iii) the 1999 CGPM recommendation that NMIs ‘continue their efforts to refine experiments that link the unit of mass to fundamental or atomic constants with a view to future redefinition of the kilogram’ and that many advances have been made in recent years in linking the mass of the international prototype to the invariant fundamental constants h and N_A,
(iv) the practicability of redefining the kelvin so that it is linked to an exact value of k, thereby eliminating the problems associated with the current definition of the kelvin based on T_TPW, which depends on the purity and isotopic composition of the water sample used,
(v) the practicability of redefining the mole so that it is linked to an exact value of N_A even when the kilogram is defined so that it is linked to an exact value of h, thereby removing the dependence of the mole on the kilogram, and
(vi) that the uncertainties of the recommended values of many fundamental constants other than h, e, k and N_A will be either eliminated or significantly reduced and that possible changes in the recommended values of those constants that are not exact resulting from future CODATA adjustments will also be significantly reduced.

It would also be useful to emphasize that the results of experiments that link the kilogram to fundamental constants have now reached levels of uncertainty such that future advances are unlikely to have any significant effect on practical mass metrology.

The new definitions should also be accompanied by an explicit presentation of the following information:

(i) the exact values of h, e, k and N_A to which the kilogram, ampere, kelvin and mole, respectively, are linked by the new definitions,
(ii) the exact values of $K_\mathrm{J} = 2e/h$ and $R_\mathrm{K} = h/e^2$, and the fact that there is no need to continue to use the conventional values $K_{\mathrm{J}-90}$ and $R_{\mathrm{K}-90}$, and also that
(iii) $m(\mathcal{K})$, the mass of the international prototype of the kilogram, is no longer exactly known and must be determined by experiment, but its value is consistent with 1 kg within an uncertainty of about two parts in 10^8,
(iv) μ_0, $\varepsilon_0 = 1/(\mu_0 c_0^2)$ and $Z_0 = (\mu_0/\varepsilon_0)^{1/2} = \mu_0 c_0$, the magnetic constant, electric constant and characteristic impedance of vacuum, are no longer exactly known and must be determined by experiment, but the value of μ_0 is consistent with $4\pi \times 10^{-7}\ \mathrm{N\,A^{-2}}$ within an uncertainty of about one part in 10^9,
(v) T_TPW, the triple point of water, is no longer exactly known and must be determined by experiment, but its value is consistent with 273.16 K within an uncertainty of about 0.25 mK, and
(vi) $M(^{12}\mathrm{C})$, the molar mass of carbon 12, is no longer exactly known and must be determined by experiment, but its value is consistent with $0.012\ \mathrm{kg\,mol^{-1}}$ within an uncertainty of less than two parts in 10^9.

The relative standard uncertainties quoted above for the values of $m(\mathcal{K})$, μ_0, $T_{\rm TPW}$ and $M(^{12}\rm C)$ that will apply after changing to the new definitions reflect what might reasonably be expected at the time the new definitions are adopted by the CGPM, presumably in 2011. They, as well as the exact values of h, e, k and $N_{\rm A}$ used in the new definitions, will be based on the CODATA 2010 recommended values of the fundamental constants. As previously indicated, the closing date for data to be considered for inclusion in the 2010 adjustment is anticipated to be 31 December 2010.

To advance the implementation of the new definitions at the time of their adoption in 2011 and thereafter, various bodies should carry out specific actions. For example, appropriate committees should draw up a *mise-en-pratique* for the new definition of the kilogram that includes recommendations concerning the various experiments that link mass to fundamental constants, as well as recommendations for the continued use of the present artefact to maintain the present excellent worldwide uniformity of mass standards. Appropriate committees should also prepare corresponding *mises-en-pratique* for realizing the ampere, kelvin and mole, including in the latter case how the molar mass of a specified entity should be calculated based on the new definition of the mole, since the molar mass of the carbon 12 atom is no longer exactly $0.012\,\rm kg\,mol^{-1}$. And these same committees should publish from time to time the best estimate together with its associated uncertainty of the mass of the international prototype expressed in terms of the new SI unit of mass, taking into account all the relevant information available at the time, as well as the best estimates together with their associated uncertainties of μ_0, ε_0, Z_0, $T_{\rm TPW}$ and $M(^{12}\rm C)$ for use in practical metrology.

The BIPM should continue to conserve and use the international prototype of the kilogram with the great care it has so ably demonstrated since 1889 so that it can provide, as part of the *mise-en-pratique* where necessary and appropriate, a practical representation of the new SI base unit of mass.

Finally, the NMIs and the BIPM should vigorously pursue their current experiments and undertake new experiments as appropriate which promise to lead to (a) a value of $m(\mathcal{K})$ together with any observed drift with time in terms of the new kilogram with $u_{\rm r}$ less than 2×10^{-8}, (b) a comparatively easy-to-use apparatus that can enable the experimental realization of the new definition of the kilogram with the appropriate uncertainty at any place at any time by anyone, (c) improved knowledge of the fine-structure constant α and hence of μ_0, ε_0 and Z_0, (d) knowledge of thermodynamic temperatures in terms of the new definition of the kelvin with uncertainties appropriate to the range such that in due course the International Temperature Scale can be dispensed with and (e) improved knowledge of $M(^{12}\rm C)$ in terms of the new definition of the mole.

4. Some consequences of the new unit definitions for present-day metrological practice and for the fundamental constants

The first part of this section is devoted to a discussion of the impact of the new definitions of the kilogram, ampere, kelvin and mole on mass metrology, electrical metrology, thermometry and chemistry, with a focus on issues about which active workers in these fields might have some concern. The benefits of redefining these units as proposed are assumed to be obvious and are only briefly touched upon. The second part of this section is devoted to a discussion of the impact of the new definitions on our knowledge of the values of the fundamental constants, possible future directions in this field, and comments on what CODATA adjustments of the values of the constants might look like after the new definitions are adopted in 2011.

4.1. *Consequences for practical metrology*

4.1.1. Impact of redefinition of the kilogram. A number of benefits to metrology will, of course, result from redefining the kilogram so that it is linked to an exactly known value of the Planck constant h, but the most significant benefit in our view is that it liberates mass metrology from an artefact-based unit. This means that different laboratories can realize the unit at will—the long-sought goal of the kilogram being realizable at any time at any place by anyone with the requisite uncertainty will now be limited only by the financial and/or human resources available at a given laboratory. Indeed, with further technical advances, it may eventually be possible to have commercially available watt balances that will enable the widespread direct realization of the new unit, in much the same way that commercially available Josephson effect voltage standards and, to a somewhat lesser extent, commercially available quantum Hall effect resistance standards have enabled the NMIs of comparatively small industrialized countries as well as large industrial laboratories to realize practical electric units based on these two effects and the conventional values $K_{\rm J-90}$ and $R_{\rm K-90}$. The anticipated relative standard uncertainty of about 2×10^{-8} in the best realization of the kilogram based on the most advanced watt balances (or, of course, on any other method) means that any variations in the calibrated values of mass standards provided by the NMIs to their customers over time can be attributed to the mass standards themselves and not to realizations of the unit. We also envision the BIPM having its own watt balance so that it can (i) monitor the mass of the international prototype of the kilogram, which we expect will play an important role in the *mise-en-pratique* of the new kilogram definition (see section 5.1), (ii) provide mass calibration services as needed to the Member States of the Metre Convention as it has so ably done since 1889 and (iii) help NMIs demonstrate, through an appropriate key comparison of travelling standards of mass, that their realizations of the new unit are consistent among themselves and have the uncertainties they claim.

4.1.2. Impact of redefinition of the ampere. As previously discussed, by linking the kilogram to an exact value of h and the ampere to an exact value of e, $K_{\rm J}$ and $R_{\rm K}$ become exactly known and the Josephson and quantum Hall effects can be used to directly realize the SI definitions of most electric units. Thus, the current practical system of conventional electric units based on these two effects and the conventional values $K_{\rm J-90}$ and $R_{\rm K-90}$ could be replaced with the SI units themselves, obviously a major advance (also see the last paragraph of section 5.2).

Although the benefits of fixing both h and e are significant, one must also recognize that the magnetic constant μ_0 (also called the permeability of vacuum), the electric constant ε_0 (also called the permittivity of vacuum) and the characteristic impedance of vacuum Z_0, which in the current SI are all exactly known constants, would become quantities that must be experimentally determined. To see this we first recall that currently

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}} \qquad \varepsilon_0 = 1/(\mu_0 c_0^2)$$
$$Z_0 = (\mu_0/\varepsilon_0)^{1/2} = \mu_0 c_0, \tag{5}$$

where, of course, $c_0 = 299\,792\,458\,\mathrm{m\,s^{-1}}$ is the speed of light in vacuum as fixed by the present definition of the metre and the value of μ_0 is fixed by the present definition of the ampere. We then recall the equation [9]

$$R_\mathrm{K} = \frac{h}{e^2} = \frac{\mu_0 c_0}{2\alpha}, \tag{6}$$

where, as before, $\alpha \approx 1/137$ is the fine-structure constant, a dimensionless quantity that is the coupling constant of the electromagnetic force. Thus, because α is simply a number determined by nature, it follows from this relation that if c_0, h and e have exactly known values, μ_0 is a quantity that must be determined by experiment. In fact, equation (6) shows that if c_0, h and e are fixed then a determination of α—for example, by equating the experimental value of and theoretical expression for the magnetic moment anomaly of the electron a_e [9]—is also a determination of μ_0, and hence ε_0 and Z_0. As discussed in section 1.3, it is reasonable to assume that u_r of the 2010 CODATA recommended value of α will be less than one part in 10^9, which implies that μ_0, ε_0 and Z_0 would be known with the same u_r. This is sufficiently small that it has no practical consequences.

4.1.3. Impact of redefinition of the kelvin.

4.1.3. Impact of redefinition of the kelvin. Because the thermometers that can be used to determine thermodynamic temperature T directly—often called 'primary thermometers'—are small in number, difficult to employ and not as precise as many practical thermometers, the quantity determined in the vast majority of present-day temperature measurements is not thermodynamic temperature but International Kelvin Temperature, T_{90}, or its Celsius-temperature equivalent, International Celsius Temperature, t_{90}, defined by $t_{90}/{}^\circ\mathrm{C} = T_{90}/\mathrm{K} - 273.15$ (see [1] and, for example, the recent review [12]). The quantities T_{90} and t_{90} are the temperatures defined by the International Temperature Scale of 1990, ITS-90, which covers the range from 0.65 K to the highest temperature measurable in practice using the Planck radiation law for monochromatic radiation. ITS-90 has recently been supplemented by the provisional low temperature scale PLTS-2000 that covers the range from 0.9 mK to 1 K and defines the corresponding new quantity T_{2000}.

In brief, the temperatures defined by ITS-90 are based on 17 equilibrium phase states of certain specified pure materials—the defining fixed points—and specified methods for interpolating between them, which include particular instruments and equations that relate measured properties of the instruments to T_{90}. However, ITS-90 is not unique, in the sense that different values of T_{90} can be obtained using (i) different interpolation equations for the same thermometer in overlapping ranges, (ii) different types of thermometers in overlapping ranges and (iii) real, that is, non-ideal, thermometers. In all the temperature ranges these differences, known as the 'non-uniqueness' and 'sub-range inconsistency' of the scale, are small—not exceeding a few tenths of a millikelvin in any part of the scale below 419 °C.

One of the defining fixed points of ITS-90 is the triple point of water T_TPW, which, according to the definition of the kelvin, is assigned the exact value 273.16 K [1]. As discussed in section 1.3, it is anticipated that by the end of 2010, the Boltzmann constant will be known with $u_\mathrm{r}(k) \approx 1 \times 10^{-6}$, which corresponds to an uncertainty of about 0.25 mK for T_TPW. However, it should be recognized that ITS-90 is a defined temperature scale for which each defining fixed point is assigned an exact value in kelvins. Hence, the value of the triple point of water on ITS-90 will remain 273.16 K, that is, $T_\mathrm{TPW-90} = 273.16\,\mathrm{K}$ exactly. The value and uncertainty of T_TPW would only need to be taken into account if for some critical reason one needed to know how well ITS-90 represents the thermodynamic temperature scale at a particular temperature or in a particular temperature range. In fact, although the consistency of T_TPW as realized by different triple point of water reference cells can be as low as 50 µK, and rather less if the isotopic composition of the water used is taken into account, the thermodynamic temperatures of all other ITS-90 defining fixed points are significantly larger [12]. Hence, the fact that T_TPW will not be exactly known but will have a standard uncertainty of 0.25 mK will have negligible practical consequences.

4.1.4. Impact of redefinition of the mole. One of the most significant benefits of redefining the mole so that it is linked to an exactly known value of the Avogadro constant N_A (assuming h, e and k also have exactly known values) is that other constants will become exactly known, namely, the Faraday constant F, molar gas constant R, Stefan–Boltzmann constant σ and molar volume of an ideal gas V_m (at a specified reference temperature and pressure), all of which have practical importance in a number of fields of chemistry and physics. For example, with an exactly known value of the Faraday constant, electrochemical measurements can be used to determine the molar mass of complex compounds with no additional uncertainty arising from a lack of knowledge of F (see section 5.1). On the other hand, the question of the preferred way of calculating molar mass does arise from the new definition of the mole, a question that we now address; a summary of our discussion may be found in table 2.

As already noted in section 2.2.4, the molar mass of an entity X is the mass of one mole of X, and it follows from the current definition of the mole that the molar mass $M(^{12}\mathrm{C})$ of the carbon 12 atom is $M(^{12}\mathrm{C}) = 0.012\,\mathrm{kg\,mol^{-1}}$. In terms of the molar mass constant M_u, defined by

$$M_\mathrm{u} = 10^{-3}\,\mathrm{kg\,mol^{-1}}, \tag{7}$$

one obtains the compact form

$$M(^{12}\mathrm{C}) = 12 M_\mathrm{u}. \tag{8}$$

The masses of atoms and molecules are most conveniently and accurately expressed not in the SI unit of mass, the

Table 2. Summary of the discussion in section 4.1.4 on the calculation of molar mass when the mole is as currently defined and when it is defined in terms of the fixed value of the Avogadro constant $\tilde{N}_A$. Here, as in the text, $m(X)$ is the mass of entity X, N_A and $M(X)$ are the Avogadro constant and molar mass of entity X, respectively, when the mole and kilogram are as currently defined, $\tilde{M}(X)$ is the molar mass of entity X when the mole is defined in terms of $\tilde{N}_A$ and the kilogram in terms of h and the molar mass factor $(1+\kappa)$ is as given in equation (17). (The unit for $M(X)$ and $\tilde{M}(X)$ is kg mol^{-1}.)

Quantity	Relationships relevant to calculating molar mass $M(X)$ when the mole is as currently defined	Relationships relevant to calculating molar mass $\tilde{M}(X)$ when the mole is defined in terms of $\tilde{N}_A$.
Unified atomic mass unit, atomic mass constant	$1\ \mathrm{u} = m_u = \dfrac{m(^{12}\mathrm{C})}{12} = \dfrac{M_u}{N_A}$	$1\ \mathrm{u} = m_u = \dfrac{m(^{12}\mathrm{C})}{12} = \dfrac{(1+\kappa)M_u}{\tilde{N}_A}$
Molar mass constant	$M_u = 10^{-3}\ \mathrm{kg\ mol^{-1}} = N_A m_u$	$M_u = 10^{-3}\ \mathrm{kg\ mol^{-1}} = \dfrac{\tilde{N}_A m_u}{(1+\kappa)}$
Relative atomic mass of entity X	$A_r(X) = \dfrac{m(X)}{m_u} = \dfrac{N_A m(X)}{M_u}$	$A_r(X) = \dfrac{m(X)}{m_u} = \dfrac{\tilde{N}_A m(X)}{(1+\kappa)M_u}$
Relative atomic mass of ^{12}C	$A_r(^{12}\mathrm{C}) = 12$	$A_r(^{12}\mathrm{C}) = 12$
Molar mass of entity X	$M(X) = N_A m(X) = A_r(X)M_u$	$\tilde{M}(X) = \tilde{N}_A m(X) = (1+\kappa)A_r(X)M_u$ $= (1+\kappa)M(X)$
Molar mass of ^{12}C	$M(^{12}\mathrm{C}) = N_A m(^{12}\mathrm{C}) = A_r(^{12}\mathrm{C})M_u$ $= 12M_u$	$\tilde{M}(^{12}\mathrm{C}) = \tilde{N}_A m(^{12}\mathrm{C}) = (1+\kappa)A_r(^{12}\mathrm{C})M_u$ $= (1+\kappa)12M_u$

kilogram, kg, but in the unified atomic mass unit u (also called the dalton, Da). A non-SI unit, the unified atomic mass unit is defined, as noted earlier, according to

$$1\ \mathrm{u} = m_u = \frac{m(^{12}\mathrm{C})}{12}, \tag{9}$$

where m_u is the atomic mass constant. Also as noted earlier, the relative atomic mass $A_r(X)$ of an entity X, which is a dimensionless quantity, is defined by the relation

$$A_r(X) = \frac{m(X)}{m_u}, \tag{10}$$

which, together with equation (9), yields $A_r(^{12}\mathrm{C}) = 12$. Equation (4) in section 2.2.4,

$$M(X) = N_A m(X), \tag{4}$$

with $X = {}^{12}\mathrm{C}$, together with equations (8) and (9), then give

$$M_u = N_A m_u. \tag{11}$$

Finally, equation (11), together with equations (4) and (10), lead to the well-known expression for the molar mass of an entity X:

$$M(X) = A_r(X)M_u. \tag{12}$$

Clearly, the new definition of the mole does not alter the basic relationship between the molar mass of an entity X, the Avogadro constant and the mass of the entity as given in equation (4), but it could affect the definitions of the molar mass constant, unified atomic mass unit, atomic mass constant and the quantity relative atomic mass, as defined in equations (7), (9) and (10). It definitely affects equations (11) and (12) because they depend on the exact relation $M(^{12}\mathrm{C}) = 12M_u$ in equation (8), which is a consequence of the current definition of the mole but is not necessarily true for the new definition. Thus, since equations (11) and (12) are not necessarily consistent with both the current definitions of the quantities that appear in them and the new definition of the mole, it is necessary to reconsider the definition of molar mass or relative atomic mass (which means the unified atomic mass unit and atomic mass constant) or even in principle the molar mass constant, in order to obtain new expressions consistent with the new definition of the mole. We address this issue in detail in the following paragraphs, where we present our preferred approach to this problem: retain the current definitions of relative atomic mass and the molar mass constant and define a new molar mass.

As just noted, the new definition of the mole does not alter the basic relationship between the molar mass of an entity X, the Avogadro constant, and the mass of the entity as given in equation (4). However, a consequence of the new definition is that, in order to be precise in the discussion of this section, equation (4) must be rewritten as $\tilde{M}(X) = \tilde{N}_A m(X)$, where $\tilde{M}(X)$ is the molar mass of X when the mole is defined so that the Avogadro constant has the exact value $\tilde{N}_A$. On the other hand, the Avogadro constant N_A in equation (11) cannot be replaced by the exact value $\tilde{N}_A$ without either changing the definition of M_u or m_u given in equations (7) and (9) or including an additional correction factor in equation (12). We recommend the latter choice and write in place of equation (11)

$$(1+\kappa)M_u = \tilde{N}_A m_u, \tag{13}$$

where

$$\tilde{N}_A = (1+\kappa)N_A, \tag{14}$$

and so the updated version of equation (12) becomes

$$\tilde{M}(X) = (1+\kappa)A_r(X)M_u. \tag{15}$$

The molar mass factor $(1+\kappa)$ can be evaluated from the definition of the Rydberg constant [9], $R_\infty = c_0\alpha^2 m_e/(2h)$, where as before α is the fine-structure constant and m_e is the mass of the electron, and the exact relation $M(^{12}\mathrm{C}) = 12M_u$ in equation (8) based on the current definition of the mole, which together yield the well-known expression

$$N_{\rm A} = \frac{c_0\alpha^2 A_{\rm r}({\rm e})M_{\rm u}}{2R_\infty h}, \qquad (16)$$

and hence

$$(1+\kappa) = \frac{\tilde{N}_{\rm A}}{N_{\rm A}} = \frac{2R_\infty \tilde{N}_{\rm A} h}{c_0\alpha^2 A_{\rm r}({\rm e})M_{\rm u}} = 1 + 0.0(0.2)\times 10^{-8}. \qquad (17)$$

The expression for R_∞ above as well as equation (16) hold even when h is assigned an exact value and is used to define the kilogram, hence it is not necessary to introduce $\tilde{h}$. On the other hand, because the Avogadro constant is now independent of the kilogram, it is necessary to introduce $\tilde{N}_{\rm A}$ since it is a different quantity from $N_{\rm A}$.

The numerical value of the molar mass factor $(1+\kappa)$ is based on the 2002 recommended values of the relevant constants under the assumption that the kilogram and mole are defined so that the Planck constant h and Avogadro constant $\tilde{N}_{\rm A}$ have their 2002 values but with no uncertainties and also on the expectation that by the time of the 2010 CODATA adjustment the relative standard uncertainty of the fine-structure constant $u_{\rm r}(\alpha)$ will be reduced to less than 10^{-9} from its present value of 3.3×10^{-9}, as discussed in section 1.3. If the latter value of $u_{\rm r}(\alpha)$ is used, the standard uncertainty of about 2×10^{-9} in equation (17) becomes 6.7×10^{-9}.

The important points concerning the molar mass factor $(1+\kappa)$ are that it will initially be equal to one when the new definition of the mole is adopted, should never deviate from unity by more than a few parts in 10^9 and, moreover, exactly cancels for molar-mass ratios in chemical reactions. This means that for all practical purposes, molar mass can continue to be calculated from the product $A_{\rm r}(X)M_{\rm u}$ because the only effect of the factor $(1+\kappa)$ would be a possible shift in the product and an additional component of uncertainty that are significantly smaller than the uncertainty of (i) practical mass measurements involving the macroscopic kilogram with which molar mass values are used and (ii) values of $A_{\rm r}(X)$ of real substances, which depend on stoichiometry, isotopic composition, impurity content, etc. In other words, its uncertainty should be sufficiently small that it can be considered negligible in calculating molar mass for use in the determination of amount of substance, since amount of substance determinations in the real world rarely, if ever, have relative standard uncertainties that approach 1×10^{-6}. Thus, for all practical chemical measurements, which is where the mole is used, molar mass should still be obtainable from equation (12).

We also see that (i) equation (13) can be written as $1\,{\rm u} = m_{\rm u} = (1+\kappa)M_{\rm u}/\tilde{N}_{\rm A}$, compared with the corresponding relation from equation (11) based on the current definition of the mole, $1\,{\rm u} = m_{\rm u} = M_{\rm u}/N_{\rm A}$, but again this a case where the factor $(1+\kappa)$ is inconsequential, and (ii) the well-known relation in equation (16) becomes

$$\tilde{N}_{\rm A} = (1+\kappa)\frac{c_0\alpha^2 A_{\rm r}({\rm e})M_{\rm u}}{2R_\infty h}. \qquad (18)$$

In summary, then, the new definition of the mole will not require any change in current metrological practice in any field.

4.2. *Impact on fundamental constants*

It is most sensible to consider the impact on our knowledge of the values of the constants of h, e, k and $N_{\rm A}$ being exactly known, which can only be described as extraordinary, all at once. If we examine the list of 2002 CODATA recommended values of the constants and energy equivalency factors given in tables XXVI and XXVIII through XXXII of [9], we find that, in addition to these four important constants, the following additional constants and factors become exactly known: h in eV s, $\hbar = h/2\pi$, $\hbar$ in eV s, $\hbar c_0$ in MeV fm, e/h, magnetic flux quantum $\Phi_0 = h/2e$, conductance quantum $G_0 = 2e^2/h$, inverse of conductance quantum G_0^{-1}, Josephson constant $K_{\rm J} = 2e/h$, von Klitzing constant $R_{\rm K} = h/e^2$, Faraday constant $F = N_{\rm A}e$, molar Planck constant $N_{\rm A}h$, $N_{\rm A}hc_0$, molar gas constant $R = kN_{\rm A}$, k in eV/K, k/h, khc_0, molar volume of ideal gas $V_{\rm m} = RT/p$ for two reference values of T and p, Loschmidt constant $n_0 = N_{\rm A}/V_{\rm m}$, Stefan–Boltzmann constant $\sigma = (2/15)\pi^5k^4/(h^3c_0^2)$, first radiation constant $c_1 = 2\pi hc_0^2$, first radiation constant for spectral radiance $c_{\rm 1L} = 2hc_0^2$, second radiation constant $c_2 = hc_0/k$ and Wien displacement law constant $b = \lambda_{\rm max}T = c_2/4.965\,114\,231\ldots$.

In addition, and also of considerable significance, the factors required for converting the value of a given quantity expressed in one of the energy-related units joules (J), kilograms (kg), inverse metres (m^{-1}), hertz (Hz), kelvins (K) or electron volts (eV) to the value of the quantity expressed in one of the other energy-related units—factors such as the numerical values of c_0^2, hc_0, h, k, e, k/hc_0, e/h, k/e, etc—would all become exactly known. Hence, the relative standard uncertainty of the value of such a quantity would be independent of the unit in which it is expressed. In particular, this means that the mass of the electron would have the same relative standard uncertainty $u_{\rm r}$ if it were expressed in either J, kg, m^{-1}, Hz, K or eV.

Although the uncertainties of other constants would not be completely eliminated, many would be significantly reduced, in particular, those involving the mass of the electron or that of other particles. From the relation $m_{\rm e} = 2hR_\infty/(c_0\alpha^2)$, and based on the current or anticipated uncertainties of R_∞ and α previously discussed, we can expect $u_{\rm r}(m_{\rm e})$ to be $<2\times10^{-9}$ compared with the 2002 CODATA value $u_{\rm r}(m_{\rm e}) = 1.7\times10^{-7}$. This means that $u_{\rm r}$ of the Bohr magneton $\mu_{\rm B} = e\hbar/2m_{\rm e}$ would also be $<2\times10^{-9}$, which is to be compared with the 2002 value $u_{\rm r}(\mu_{\rm B}) = 8.6\times10^{-8}$. Since $u_{\rm r}$ of the ratio $m_{\rm p}/m_{\rm e} = A_{\rm r}({\rm p})/A_{\rm r}({\rm e})$, where p stands for proton, is currently 4.6×10^{-10}, we can also expect $u_{\rm r}(m_{\rm p})$ to be somewhat less than 2×10^{-9} and that of the nuclear magneton $\mu_{\rm N} = e\hbar/2m_{\rm p}$ to be the same. Further, because in general $m_X/m_{\rm e} = A_{\rm r}(X)/A_{\rm r}({\rm e})$, and the relative atomic masses $A_{\rm r}(X)$ of many particles and atoms have $u_{\rm r} \approx 10^{-10}$, their masses in kilograms will be known with the same $u_{\rm r}$ as that of $m_{\rm e}$.

A question that will certainly be raised is the following: Will there still be a need for research in the field of fundamental constants and what will be the mission of the CODATA Task Group on Fundamental Constants after the new units are adopted and h, e, k and $N_{\rm A}$ become exactly known? A brief examination shows that much will still need to be done. For example, measurements and calculations that will lead to reduced uncertainties for α, R_∞, $A_{\rm r}({\rm e})$ and the relative atomic masses of other particles will still be important for advancing our knowledge of the values of the constants, as can be seen even from the brief discussion in this section of the paper. And then there is the possibility of determining R_∞ with a sufficiently small uncertainty that the second can be redefined so that it is linked to an exact value of R_∞. This will require, for example, improved measurements of transition frequencies in hydrogen (H) and deuterium (D) relative to various transition frequencies in other atoms and ions and to one another, together with improved calculations of various contributions to the theoretical expressions of H and D energy levels [9, 22]. If successful, it will allow (i) the creation of a *mise-en-pratique* for the realization of the second using the frequencies of a number of different radiations as is currently the case for the realization of the metre as well as (ii) the calculation of H and D transition frequencies with the smallest possible uncertainties so that hydrogen and deuterium can be used as a source of a broad range of radiations of known frequency.

It should also be recognized that certain experiments previously aimed at determining the value of a fundamental constant with a reduced uncertainty now become experiments aimed at the realization of a unit. For example, watt-balance experiments will no longer be carried out to determine h but rather to realize the kilogram, and molar gas constant or Boltzmann constant experiments will no longer be carried out to determine R or k but rather to realize the kelvin.

With regard to future least-squares adjustments of the constants, although it is certainly true that after the formal adoption of the new unit definitions, presumed to be in 2011, many constants will have fixed values, the majority will not. Thus, a set of recommended values of the constants for international use throughout science and technology that reflects current knowledge at a given point in time will still be in demand. Moreover, best values of quantities relevant to practical metrology will result from such adjustments, for example, the mass $m(\mathcal{K})$ of the international prototype, μ_0, ε_0 and Z_0, the triple point of water T_{TPW} (and/or other ITS-90 fixed points) and the molar mass factor $(1+\kappa) = 2R_\infty \tilde{N}_{\mathrm{A}} h/[c_0\alpha^2 A_{\mathrm{r}}(\mathrm{e})M_{\mathrm{u}}]$.

Finally, we believe that a very positive effect of fixing the values of h, e, k and N_{A} and the accompanying elimination or reduction in uncertainty of most other constants will be the greatly improved clarity in the links among fundamental constants, as well as a highlighting of those areas of physics where important advances can be made through improved theory and experiment.

5. Some considerations for the *mises-en-pratique* of the new unit definitions

In its Recommendation 1 (CI-2005), the CIPM addressed the preparation of detailed guidelines on how the new definitions of the kilogram, ampere, kelvin and mole should be realized in practice. Such detailed guidelines are known as *mises-en-pratique* of unit definitions. The bodies best suited to this task are the relevant CCs of the CIPM and their working groups, namely, the CC for Mass and Related Quantities (CCM), the CC for Electricity and Magnetism (CCEM), the CC for Thermometry (CCT) and the CC for Amount of Substance–Metrology in Chemistry (CCQM), in recognition of the fact that such efforts should undoubtedly be carried out by experts actively working in the fields of mass metrology, electrical metrology, thermometry and chemistry, particularly analytical chemistry. Preliminary discussions at some of the relevant CCs have, however, already indicated the broad content of each individual *mise-en-pratique* and we build upon these in what follows. The aim of this section is, therefore, simply to inform those not familiar with the content of a *mise-en-pratique* about the issues involved. We emphasize that the CIPM will be the body that finally adopts each *mise-en-pratique* on the advice of the relevant CC.

5.1. Kilogram

In practice, and depending on the institution and the uncertainty desired, the new definition of the kilogram would most likely be realized by one of three general methods: (i) an experiment that directly links an unknown macroscopic standard of mass to the exactly known values of fundamental constants, (ii) a similar experiment but one in which the values of some of the fundamental constants are not exactly known or (iii) a comparison of an unknown macroscopic standard of mass with a known, specified macroscopic standard of mass, whose mass may itself be determined by one of the methods (i) or (ii). The first category is currently limited to the moving-coil watt balance (or an equivalent apparatus that relates mechanical power or energy to electrical power or energy), which enables a mass standard to be related to the exactly known Planck constant and quantities directly measurable in the experiment such as length, frequency, time and resistance ratio.

The second category includes the XRCD experiment, which enables the mass of a nearly perfect single crystal silicon sphere of mass of about 1 kg to be related to the exactly known Avogadro constant as well as to the ratio of two lengths, but also requires knowing the molar mass of the sphere's silicon atoms and hence the use of the inexactly known molar mass factor $(1+\kappa)$. In this category as well is the classic electrochemical Faraday constant experiment [9, 23] (including the new 'vacuum' version underway at PTB [24]) that enables the macroscopic mass of a large number of atoms to be related to the exactly known Faraday constant and, again, quantities directly measurable in the experiment such as frequency, time and resistance ratio. But like the XRCD experiment, it also requires knowing the molar mass of some species of atom and hence the use of the inexactly known molar mass factor $(1+\kappa)$.

The relevant equations for these three experiments—moving-coil watt balance, XRCD method and Faraday constant—may be written as follows, where it is now understood that N_{A} is the fixed value used to define the mole (i.e. the tilde used in section 4.1.4 is dropped):

$$m_x = \frac{UI}{gv} = h\frac{K_{\mathrm{J}}^2 R_{\mathrm{K}} UI}{4gv} = h\frac{\beta}{4gv}, \tag{19}$$

$$m_{\mathrm{s}}(\mathrm{Si}) = \frac{\pi(1+\kappa)A_{\mathrm{r}}(\mathrm{Si})M_{\mathrm{u}}}{12\sqrt{2}N_{\mathrm{A}}}\left(\frac{d_{\mathrm{s}}(\mathrm{Si})}{d_{220}(\mathrm{Si})}\right)^3, \tag{20}$$

$$m_{\mathrm{d}}(X) = \frac{1}{F}\,\frac{(1+\kappa)A_{\mathrm{r}}(X)M_{\mathrm{u}}It}{z}. \tag{21}$$

Since the constants h, N_{A}, F, K_{J} and R_{K} enter into these experiments, it would certainly be appropriate for their exactly known values to be included in the *mise-en-pratique*. In equation (19), m_x is the unknown mass of a mass standard, U is the induced voltage in the coil in the part of the experiment in which the coil is moved vertically in a radial magnetic flux density B with velocity v, I is the current in the coil in the weighing part of the experiment in which the force $m_x g$ on the coil is balanced by the electrical force on the coil due to I and B, g is the local acceleration of free-fall and β, the unit of which is s^{-2}, represents the experimental quantities frequency and resistance ratio required to measure U and I in terms of the exactly known values of K_J and R_K.

In equation (20), $m_\mathrm{s}(\mathrm{Si})$ is the unknown mass of a nearly perfect single crystal sphere of silicon, $A_\mathrm{r}(\mathrm{Si})$ is the relative atomic mass of the silicon atoms of which the sphere is composed, $d_\mathrm{s}(\mathrm{Si})$ is the mean diameter of the sphere and $d_{220}(\mathrm{Si})$ is the $\{220\}$ lattice spacing of the silicon crystal.

In equation (21), $m_\mathrm{d}(X)$ is the mass after neutralization of the deposited (or collected) charged entities X of valence z, $A_\mathrm{r}(X)$ is their relative atomic mass, I is the current due to the flow of the charged entities, and t is the duration of charge flow. The current I is, of course, measured using the Josephson and quantum Hall effects and the exactly known values of K_J and R_K, which only requires the measurement of frequency and resistance ratio.

The third category, at least in the years just following the new definition, would mostly be through comparisons of standards with the international prototype that will still be kept and used at the BIPM. This category will be for those NMIs that choose not to operate one of the experiments mentioned above and will, of course, provide a cost effective way of maintaining mass standards at the highest level. At the time of adoption of the new definition of the kilogram, presumably in 2011, the best estimate of the mass of the international prototype $m(\mathcal{K})$ in terms of the new kilogram will be 1 kg. Its relative standard uncertainty will be the same as the 2010 CODATA recommended value of h, which we assume will be about 2×10^{-8} (see section 1.3). However, future measurements with the most precise watt balances or XRCD apparatus may show that its mass has drifted and this would be corrected, probably by including the new data in subsequent CODATA adjustments and taking $m(\mathcal{K})$ as an adjusted constant (variable). Consequently, one would expect the *mise-en-pratique* for the kilogram to be amended or revised to reflect the change. At any time, the mass of the international prototype could be linked to the definition through comparisons with mass standards used in the most accurate watt balances or XRCD apparatus through a special key comparison. The value and uncertainty of its mass would then be that resulting from the key comparison.

5.2. Ampere

Although many NMIs as well as industrial laboratories have long experience in realizing the practical electric units of voltage, resistance, current, etc based on the Josephson and quantum Hall effects and the conventional values of the Josephson and von Klitzing constants $K_{\mathrm{J}-90}$ and $R_{\mathrm{K}-90}$ adopted by the CIPM for use throughout the world starting 1 January 1990 [1], it probably would still be useful to include all of the following in the *mise-en-pratique*: (i) derivation and presentation of the exact value of h to which the kilogram is linked, (ii) derivation and presentation of the exact value of e to which the ampere is linked, (iii) calculation and presentation of the exact values of the Josephson and von Klitzing constants $K_\mathrm{J} = 2e/h$ and $R_\mathrm{K} = h/e^2$ implied by the exact values of e and h and a recommendation for the truncated values that should be used by workers in the field (that is, specific recommended values to replace $K_{\mathrm{J}-90} = 483\,594.9\ \mathrm{GHz\,V^{-1}}$ and $R_{\mathrm{K}-90} = 25\,812.807\ \Omega$), (iv) a brief review of the basic Josephson and quantum Hall effect relations for obtaining a known Josephson voltage U_J and a known quantized Hall resistance (QHR) $R(i)$, namely, $U_\mathrm{J}(n) = nf/K_\mathrm{J}$ and $R(i) = R_\mathrm{K}/i$ (the integer n is the number of the constant-voltage current-step induced by microwave radiation of frequency f applied to a Josephson device and the integer i is the number of the QHR plateau used), (v) a brief review of single electron tunnelling (SET) devices that may possibly be used to realize the ampere at very low current levels and the basic SET equation $I = ef$ (where f is the frequency of the signal applied to the SET device), (vi) the CCEM's 'Revised guidelines for reliable dc measurements of the QHR' [25], (vii) possible similar guidelines for the use of Josephson array voltage standard devices, (viii) possible general guidance on realizing the farad and henry from $R(i)$ and (ix) the current best value and associated uncertainty of the magnetic constant μ_0, electric constant $\varepsilon_0 = 1/(\mu_0 c^2)$ and characteristic impedance of vacuum $Z_0 = \mu_0 c$. As shown in section 4.1.2, at the time of the adoption of the new unit definitions, anticipated to be 2011, the best estimate of μ_0 will be $4\pi \times 10^{-7}\ \mathrm{N\,A^{-2}}$ with an associated u_r that will be the same as that of the 2010 recommended value of the fine-structure constant α, which we assume will be less than one part in 10^9 based on the discussion in section 1.3.

Of course, in analogy with the case of $m(\mathcal{K})$ discussed in section 5.1, the best estimated values of μ_0, ε_0, Z_0 and their uncertainties would be expected to change somewhat in subsequent adjustments due to a change in the adjusted value of α resulting from new input data, and thus one would expect the *mise-en-pratique* for the ampere to be amended or revised to reflect the changes.

The most significant problem that the electrical metrology community will likely have to deal with is the slight changes in the values of the electrical units disseminated by the NMIs that will result from the implementation of the new kilogram and ampere definitions. The cause of the changes is the slight difference between the conventional values $K_{\mathrm{J}-90}$ and $R_{\mathrm{K}-90}$ and the exact SI values—for example, based on the 2002 CODATA adjustment [9], we have $K_\mathrm{J} = K_{\mathrm{J}-90}[1 - 4.3(8.5) \times 10^{-8}]$ and $R_\mathrm{K} = R_{\mathrm{K}-90}[1 + 1.74(33) \times 10^{-8}]$. However, the differences should be sufficiently small that the changes will have an inconsequential effect on the vast majority of practical electrical measurements.

5.3. Kelvin

The *mise-en-pratique* of the kelvin that reflects its new definition linking it to an exact value of the Boltzmann constant may be the easiest of the four to prepare. This is because the CC for Thermometry, as proposed in its Recommendation T 3 (2005) to the CIPM, has already recommended 'the creation of a *mise-en-pratique* of the definition of the kelvin containing, in due course, recommendations concerning the direct determination of thermodynamic temperature, the text of the ITS-90, the text of the PLTS-2000, a Technical Annex of material deemed essential for the unambiguous realization of both the ITS-90 and the PLTS-2000 and a section discussing the differences $T - T_{90}$ and $T - T_{2000}$ together with their uncertainties' and 'approval by the CIPM of the text entitled 'Technical Annex for the *mise-en-pratique* of the definition of the kelvin', adopted by the CCT at its 23rd meeting, as initial entry to the Technical Annex.' This recommendation was approved by the CIPM at its 94th meeting in October 2005 [10].

As first discussed in section 1.3, the best estimate of the triple point of water T_{TPW} in terms of the newly defined kelvin at the time of adoption of the new units in 2011 will be 273.16 K, its current exact value, but with u_{r} equal to that of the 2010 CODATA recommended value of k, which we assume will be about 1×10^{-6}, corresponding to about 0.25 mK for T_{TPW}. However, as noted in section 1.3 and discussed further in section 4.1.3, the existence of this uncertainty will have negligible consequences.

As in the case of $m(\mathcal{K})$, μ_0, ε_0 and Z_0 discussed in sections 5.1 and 5.2, it should be recognized that the best estimate of T_{TPW} and its uncertainty would be expected to change somewhat as new data on the relationship between T_{TPW} and k became available, and one would thus expect ITS-90 and hence the *mise-en-pratique* of the kelvin to be amended or revised to reflect the change.

5.4. Mole

Precise measurements of amount of substance for a sample in a chemical laboratory are generally made by weighing the sample and then using the molar mass of the material to convert the mass into amount of substance. Such measurements are rarely—if ever—made with an uncertainty approaching parts per million, and nothing proposed in this paper will have any effect on such measurements or the way they are done. The current methods of carrying out chemical measurements so that the results can be expressed in the SI unit of amount of substance, the mole, can still be used with the new definition of the mole. As discussed in section 4.1.4, if our recommended approach to calculating molar masses from relative atomic masses is followed, the only change the new definition introduces is in fact the calculation of molar mass from $A_{\mathrm{r}}(X)$. The new relation is given in equation (15), $\tilde{M}(X) = (1 + \kappa)A_{\mathrm{r}}(X)M_{\mathrm{u}}$, which is to be compared with the old relation in equation (12), $M(X) = A_{\mathrm{r}}(X)M_{\mathrm{u}}$, where the molar mass factor $(1+\kappa)$ is as given in equation (17). However, because it is essentially equal to one (within a few parts in 10^9 at worst) and its uncertainty is $<2 \times 10^{-9}$, in practice it can be ignored. We believe that all this should be carefully explained in the *mise-en-pratique* of the mole, that the 2010 CODATA recommended value of the molar mass factor $(1+\kappa)$ and its associated uncertainty should be included with the explanation, and that a few practical molar mass calculations which explicitly show why the factor is negligible should also be given.

Once again, as in the case of $m(\mathcal{K})$, μ_0, ε_0, Z_0 and T_{TPW} discussed in sections 5.1–5.3, the best estimated value of $(1+\kappa)$ and its uncertainty would be expected to change somewhat in subsequent CODATA adjustments due to slight shifts in the adjusted values of R_∞, α and $A_{\mathrm{r}}(\mathrm{e})$ arising from new input data. Although in principle the *mise-en-pratique* for the mole should be amended or revised to reflect the change, one may safely assume that any change would be insignificant in terms of practical measurements and calculations of amount of substance.

6. Conclusion

Redefining the SI base units kilogram, ampere, kelvin and mole by linking them to exactly known values of the Planck constant h, elementary charge e, Boltzmann constant k and Avogadro constant N_{A}, respectively, as we have proposed in this paper, would implement CIPM Recommendation 1 (CI-2005) in a way that would be profoundly beneficial to both metrology and our knowledge of the values of the fundamental physical constants in SI units, or more generally, to quantum physics. Our suggestion that one might go even further and fix the magnitudes of the SI units simply by adopting fixed values for a set of constants seems to us a logical extension of current thinking and we offer it for discussion. It is not unreasonable to expect that the results from the several relevant fundamental constant experiments currently underway will be both satisfactory and available by the end of 2010, which would enable the new definitions to be formally adopted by the 24th CGPM in 2011. Such adoption will ensure that the SI can meet the future needs of both practical metrology and quantum physics and hence that the SI can continue to serve as a common measurement language for intelligible communication between these two important communities.

Appendix. CIPM and CCU Recommendations

The first recommendation is that adopted by the CIPM at its 94th meeting in October 2005 [10] in response to the second recommendation, which was submitted to the CIPM by the CCU as a result of the CCU's 17th meeting held in June/July 2005.

RECOMMENDATION OF THE INTERNATIONAL COMMITTEE FOR WEIGHTS AND MEASURES

Preparative steps towards new definitions of the kilogram, the ampere, the kelvin and the mole in terms of fundamental constants

RECOMMENDATION 1 (CI-2005)

The International Committee for Weights and Measures (CIPM),
considering

- Resolution 7 of the 21st General Conference on Weights and Measures (CGPM), 1999, concerning a future definition of the kilogram;
- the recent, 2005, Recommendations of the Consultative Committee for Mass and Related Quantities (CCM), the Consultative Committee for Electricity and

Magnetism (CCEM), the Consultative Committee for Amount of Substance–Metrology in Chemistry (CCQM) and the Consultative Committee for Thermometry (CCT) concerning proposals for and matters related to changes in the definitions of the kilogram, the ampere and the kelvin;
- the Recommendation of the CCU [Recommendation U 1 (2005)] which brings together all of the major points of these other recommendations and which requests that the CIPM:
  - approve in principle the preparation of new definitions and *mises en pratique* of the kilogram, the ampere and the kelvin so that if the results of experimental measurements over the next few years are indeed acceptable, all having been agreed with the various Consultative Committees and other relevant bodies, the CIPM can prepare proposals to be put to Member States of the Metre Convention in time for possible adoption by the 24th CGPM in 2011;
  - give consideration to the possibility of redefining, at the same time, the mole in terms of a fixed value of the Avogadro constant;
  - prepare a Draft Resolution that may be put to the 23rd CGPM in 2007 to alert Member States to these activities;
  - further encourage National Metrology Institutes to pursue national funding to support continued relevant research in order to facilitate the changes suggested here and improve our knowledge of the relevant fundamental constants, with a view to further improvement in the International System of Units;
- the need for careful consideration to be given to both the form and content of possible new definitions of these units, not only individually but also taken as an ensemble;

approves, in principle, the preparation of the new definitions, as requested by the CCU in its Recommendation cited above;

invites all Consultative Committees

- particularly the CCM, CCEM, CCQM and CCT, to consider the implications of changing the definitions of the above-mentioned base units of the SI, and to submit a report to the CIPM not later than June 2007;
- to monitor closely the results of new experiments relevant to the possible new definitions, to identify necessary conditions to be met before proceeding with changing the definitions, and to consider, in particular, the alternative ways of redefining the above mentioned units;
- to solicit input from the wider scientific and technical community on this important matter;

recommends that National Metrology Institutes

- should pursue vigorously their work presently underway aimed at providing the best possible values of the fundamental constants needed for the redefinitions now being considered;
- should prepare for the long term maintenance of those experiments that will, in due course, be necessary for the practical realization of the new definitions.

RECOMMENDATION OF THE CCU TO THE CIPM

On possible changes to the definitions of the kilogram, the ampere, the kelvin and the mole

RECOMMENDATION U1 (2005)

The CCU,
considering

- the responsibilities of the CCU, namely:
  - those given to it at its creation in 1964 by the CIPM concerning the development of the SI,
  - its responsibility for the drawing up of successive editions of the SI brochure,
  - the further responsibility of giving advice to the CIPM on matters related to units of measurement;
- the importance of taking a broad and profound view of the SI to ensure that it meets the needs of all users while at the same time ensuring that it reflects advances in science and in the understanding of the structure of physics;
- the great improvements that have taken place in the accuracy of our knowledge of the values of most of the fundamental constants of physics since the last change in the definition of a base unit in 1983, which fixed the value of the speed of light in vacuum;
- the impact on metrology of the application of the Josephson and quantum Hall effects;
- the consensus that now exists on the desirability of finding ways of defining all of the base units of the SI in terms of fundamental physical constants so that they are universal, permanent and invariant in time;
- Resolution 7 of the 21st CGPM, 1999, concerning a future definition of the kilogram;
- the recent (2005) recommendations from the CCM, the CCEM, and the CCT to the CIPM concerning possible redefinitions of the kilogram to fix, for example, the Planck constant, the ampere to fix the elementary charge and the kelvin to fix the Boltzmann constant, and also from the CCQM in relation to the interests of the chemical community;
- the recent recommendation to the CCU from the CODATA Task Group on Fundamental Constants supporting the redefinitions above, and also on redefining at the same time the mole in terms of a fixed value of the Avogadro constant;
- the broad view that has emerged from discussions at these meetings of Consultative Committees and the CODATA Task Group, that if changes do take place in the definitions of the kilogram, the ampere and the kelvin, they should all take place at the same time;
- that further experimental results are essential, as noted by the Consultative Committees in their Recommendations cited above, before redefinition of the base units could be implemented;
- that before such important changes are made to the definitions of base units of the SI, wide publicity must be given to the draft proposals so that the opinion of the broad scientific and other user communities, not directly touched by the Consultative Committee structure of the Metre Convention, can be obtained and taken into account;

requests that

- the CIPM approve in principle the preparation of new definitions and *mises-en-pratique* of the kilogram, the ampere and the kelvin so that if the results of experimental measurements are indeed acceptable, all having been agreed with the various Consultative Committees and other relevant bodies, the CIPM can prepare proposals to be put to Member Governments of the Metre Convention in time for possible adoption by the 24th CGPM in 2011;
- the CIPM give consideration to the possibility of redefining, at the same time, the mole in terms of a fixed value of the Avogadro constant;
- the CIPM prepare a Resolution that may be put to the 23rd CGPM in 2007 to alert member states to these activities;
- the CIPM further encourage NMIs to pursue national funding to support continued relevant research in order to facilitate the changes suggested above and improve our knowledge of the relevant fundamental constants, with a view to further improvement in the International System of Units.

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NIST is part of the Technology Administration, US Department of Commerce

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